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S.H. 1829

A

KEY

TO

MR. KEITH'S
COMPLETE MEASURER;

OR, THE

Whole Art of Measuring:

CONTAINING

THE SOLUTIONS TO ALL THE QUESTIONS IN
THAT WORK.

**WITH REFERENCES TO THE PROBLEMS, RULES, & NOTES,
BY WHICH THE SOLUTIONS ARE OBTAINED.**

THE WHOLE TOGETHER FORMING A COMPREHENSIVE SYSTEM OF
MENSURATION, BOTH IN THEORY AND PRACTICE, AND EX-
PLAINED IN AN EASY AND FAMILIAR MANNER, SO THAT
THE STUDENT MAY, IN A SHORT TIME, AND WITH
VERY LITTLE LABOUR, OBTAIN A COMPETENT
KNOWLEDGE OF MENSURATION.

By **SAMUEL MAYNARD,**

Master of the Mathematical and Commercial Academy, No. 16,
Clarendon Square, Sommers Town, London.

" Whatsoever is difficult, this teaches to overcome."—Ovid.

BODLEIAN

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PREFACE.

If an art or science is to be valued in proportion to its utility and importance in the actual concerns of human affairs, Mensuration, considered in its utmost extent, will stand pre-eminently distinguished.

Mensuration is of the highest importance in many affairs that are absolutely necessary, and in every branch of the mathematics: by it sciences are established, and commerce is conducted; by its aid we manage our business, and inform ourselves of the wonderful operations of nature; by it we measure the heavens and the earth,—for it is by the aid of Mensuration that astronomers are put into a way of making their observations, coming at the knowledge of the extent of the heavenly bodies, measures of seasons, of years, and of ages; and that geographers present at once to our view the magnitude of the whole earth, the vast extent of the seas, the divisions of empires, kingdoms, and provinces. It is from this that the architect takes his just measures for the structure of public buildings, as well as of private houses. By its help engineers conduct all their works; take the situation and plan of places; measure their distances from each other, and carry their measure into places that are only accessible to the eye. By this means we estimate the capacities of all vessels, and bulks of all bodies, gauge our liquors, measure our lands, and the works of artificers, and buy and sell an infinite variety of things useful in life.

There is scarcely a writer on the different branches of education, who has not expressly recommended the study of Mensuration. An able and complete treatise on this subject was published a few years ago by Mr. KEITH, entitled, "*The Complete Measurer*; *or the whole Art of Measuring.*" This work contains the substance of Hawney's "Measurer," (a book well known for a century past,) with the addition of about 453 new questions, (*see* Mr. Keith's Preface), with the demonstration of the Rules. It contains also a greater number of questions, and more useful matter, than any other work that has hitherto appeared; and in order that the student may judge for himself, I shall here state the several parts of which Mr. Keith's work consists:—

Part I. treats of Practical Geometry.

Part II. treats of the Mensuration of Superficies, and includes all the most useful Rules for finding the Areas of right-lined and circular Figures, with the Mensuration of the Conic Sections, the general Properties of the Ellipsis, the Parabola, and the Hyperbola; also, to find the Area of Curves by equidistant Ordinates.

Part III. treats of the Mensuration of Solids that are bounded by right or circular lines, *viz.*—Cubes, Prisms, Cylinders, Pyramids, the Sphere, the circular Spindle, &c., with the Conic Sections in general; also of the Five Regular Bodies, the Specific Gravity of Solids, and the method of ascertaining the weight and dimensions of Balls and Shells.

Part IV. treats of the description of the Carpenter's Rule, Mensuration of Timber and of Artificers' Work, *viz.* Carpenters' and Joiners' Work, flooring, partitioning, roofing, and wainscotting; Bricklayers' Work, tiling or slating, walling, and chimnies; Plasterers, Painters, Glaziers, Plumbers, Masons, and Pavers' Work; with the method of ascertaining the dimensions of Vaulted and Arched Roofs, Domes, Saloons, &c. &c.

Part V. treats of the method of Gauging all kinds of Vessels used by Maltsters, Brewers, Distillers, Wine Merchants, Victuallers, &c., such as cisterns, coppers, vats, stills, casks, &c.

Part VI. treats of Land Surveying; explaining the use of the Instruments, the methods of surveying, of planning, of computing the contents, of reducing plans, of dividing the ground, of entering the field notes in the field book, and of surveying a large estate; also to survey a town or city: and likewise a large collection of useful Miscellaneous Questions in Mensuration.

The great reputation Mr. Keith's Mensuration has obtained among mathematicians and experienced teachers, renders it unnecessary, and therefore superfluous, for the Author, in this place, to say any thing further in its commendation. He therefore is of opinion, that he cannot do a more acceptable service to the public, or pay a more respectful tribute to the persevering industry, talents, and attainments, of its deceased author, than writing a Key to this very useful and important work. The solutions in this Key are in general obtained by the shortest, most simple, and most approved methods the Author could devise; but in doing this, he has closely followed Mr. Keith's Rules, and wherever a solution could be illustrated, or made more easy by a diagram, it has been introduced. He also wishes further to notice, that under the article "Gauging," Rules are given for reducing the Old Standard into the Imperial, and conversely, to reduce the Imperial Standard into the Old. It is hoped the attentive reader will find fewer difficulties in this work than in any other production of a similar kind. Great plainness has been studied, and the greatest pains have been taken to express every thing in the clearest manner possible; and the Author has spared neither labour nor trouble to render it generally useful: so that the science of Mensuration is laid open at once to any one desirous of obtaining it; and it is presumed, that should the student be only acquainted with common arithmetic, without any previous knowledge of the subject, he will, by the assistance of this Key alone, be enabled to solve any question in Mr. Keith's Mensuration. In working each question, particular care has been taken to give explicit reference to the identical Rules and Notes by which the solution is obtained.

It will be perceived in this publication, that the Author has endeavoured, and he hopes with success, to treat the subjects in a manner better adapted to the improved state of knowledge in the present times, than they have usually been treated in any work hitherto written on the same subject. The *reason* is made to go hand in hand with the *operation*; the object and utility of every Problem is shortly, but completely explained; and thus a perfect knowledge of every problem is secured to the reader.

The Signs and Characters used in this Key, are placed at the beginning, and the reader should acquaint himself with them before he peruses the work.

The chief design of the Author in writing this Key was to remedy the inconvenience he had experienced in his own school, so much of his time being consumed in going through the whole of a solution, in order to detect the errors which his pupils might have made. How far the Author has succeeded in facilitating the labour of youthful studies, must be left to the judgment of his candid readers. But should he be so successful as to render the subject plain and easy to the student, and thereby to diminish the labour of the master, his expectations will be fully answered, and he shall feel the pleasing satisfaction of having done his utmost to promote those studies, which are acknowledged on all hands to be so highly beneficial to mankind.

SAMUEL MAYNARD.

16, *Clarendon Square, Sommers Town, London,*
December 25, 1828.

DIRECTIONS FOR THE BINDER.

The first 15 Diagrams to face the Title.
Figure 16, to face .... Page 133
Figure 17, to face .... Page 110

CONTENTS.

Table of Composite Numbers x
Explanation of the Characters used in this Work .. xi
Mensuration of Superficies 1
Of the Conic Sections 26
Mensuration of Solids 41
Of the Five Regular Bodies 71
Of the Specific Gravity of Solids............. 73
Of the Weight and Dimensions of Balls and Shells 82
Mensuration of Timber and of Artificers' Work .. 84
Of Carpenters' and Joiners' Work 90
Of Bricklayers' Work 95
Of Plasterers' Work......................... 98
Of Painters' Work 101
Of Glaziers' Work 102
Of Plumbers' Work .... *ib.*
Of Masons' Work 103
Of Pavers' Work .... *ib.*
Of Vaulted and Arched Roofs, &c. 104
Gauging 111
Land Surveying.............................. 129
Miscellaneous Questions 138
Table of Lineal Measure 153
—— of Square Measure....................... *ib.*
—— of Cubic or Solid Measure............... 154
—— of Old Wine Measure reduced to Imperial.. 155
—— of Old Ale and Beer Measure reduced to Imperial.......................... *ib.*
—— of Old Dry Measure reduced to Imperial .. 156

A
TABLE
OF
COMPOSITE NUMBERS,
CONSISTING OF
THREE COMPONENT PARTS,
WITHIN THE LIMITS OF THE MULTIPLICATION TABLE.

No.	Comp. Parts.	No.	Comp. Parts.	No.	Comp. Parts.	No.	Comp. Parts.
75	5 . 5 . 3	245	7 . 7 . 5	448	8 . 8 . 7	768	12 . 8 . 8
98	7 . 7 . 2	250	10 . 5 . 5	450	10 . 9 . 5	770	11 . 10 . 7
105	7 . 5 . 3	252	12 . 7 . 3	462	11 . 7 . 6	792	12 . 11 . 6
112	8 . 7 . 2	256	8 . 8 . 4	480	12 . 10 . 4	800	10 . 10 . 8
125	5 . 5 . 5	264	11 . 6 . 4	484	11 . 11 . 4	810	10 . 9 . 9
126	9 . 7 . 2	270	10 . 9 . 3	486	9 . 9 . 6	840	12 . 10 . 7
128	8 . 8 . 2	275	11 . 5 . 5	490	10 . 7 . 7	847	11 . 11 . 7
135	9 . 5 . 3	280	10 . 7 . 4	495	11 . 9 . 5	864	12 . 9 . 8
140	10 . 7 . 2	288	12 . 12 . 2	500	10 . 10 . 5	880	11 . 10 . 8
147	7 . 7 . 3	297	11 . 9 . 3	504	9 . 8 . 7	891	11 . 9 . 9
150	10 . 5 . 3	300	10 . 10 . 3	512	8 . 8 . 8	900	10 . 10 . 9
154	11 . 7 . 2	308	11 . 7 . 4	528	12 . 11 . 4	924	12 . 11 . 7
160	10 . 8 . 2	315	9 . 7 . 5	539	11 . 7 . 7	960	12 . 10 . 8
162	9 . 9 . 2	320	10 . 8 . 4	540	10 . 9 . 6	968	11 . 11 . 8
165	11 . 5 . 3	324	9 . 9 . 4	550	11 . 10 . 5	972	12 . 9 . 9
168	8 . 7 . 3	330	11 . 10 . 3	560	10 . 8 . 7	990	11 . 10 . 9
175	7 . 5 . 5	336	12 . 7 . 4	567	9 . 9 . 7	1000	10 . 10 . 10
176	11 . 8 . 2	343	7 . 7 . 7	576	12 . 12 . 4	1008	12 . 12 . 7
180	10 . 9 . 2	350	10 . 7 . 5	594	11 9 . 6	1056	12 . 11 . 8
189	9 . 7 . 3	352	11 . 8 . 4	600	12 . 10 . 5	1080	12 . 10 . 9
192	12 . 8 . 2	360	12 . 10 . 3	605	11 . 11 . 5	1089	11 . 11 . 9
196	7 . 7 . 4	363	11 . 11 . 3	616	11 . 8 . 7	1100	11 . 10 . 10
198	11 . 9 . 2	378	9 . 7 . 6	630	10 . 9 . 7	1152	12 . 12 . 8
200	10 . 10 . 2	384	8 . 8 . 6	640	10 . 8 . 8	1188	12 . 11 . 9
210	10 . 7 . 3	385	11 . 7 . 5	648	9 . 9 . 8	1200	12 . 10 . 10
216	12 . 9 . 2	392	8 . 7 . 7	660	11 . 10 . 6	1210	11 . 11 . 10
220	11 . 10 . 2	396	11 . 9 . 4	672	12 . 8 . 7	1296	12 . 12 . 9
224	8 . 7 . 4	400	10 . 10 . 4	693	11 . 9 . 7	1320	12 . 11 . 10
225	9 . 5 . 5	405	9 . 9 . 5	700	10 . 10 . 7	1331	11 . 11 . 11
231	11 . 7 . 3	420	10 . 7 . 6	720	12 . 10 . 6	1440	12 . 12 . 10
240	12 . 10 . 2	432	12 . 9 . 4	726	11 . 11 . 6	1452	12 . 11 . 11
242	11 . 11 . 2	440	11 . 10 . 4	729	9 . 9 . 9	1584	12 . 12 . 11
243	9 . 9 . 3	441	9 . 7 . 7	756	12 . 9 . 7	1728	12 . 12 . 12

EXPLANATION OF THE CHARACTERS USED IN THIS WORK.

$+$ denotes *plus*, or *more*. The sign of *addition*, signifying that the numbers between which it is placed are to be added together. Thus, $9 + 6$ denotes that 6 is to be added to 9. Geometrical lines are generally represented by capital letters. Thus, $AB + CD$ signifies that the line AB is to be added to the line CD.

$-$ denotes *minus*, or *less*. The sign of *subtraction*, signifying that the latter of the two numbers between which it is placed is to be taken from the former. Thus, $5 - 3$ denotes that 3 is to be taken from 5. In geometrical lines also, $AB - CD$ signifies that the line CD is to be subtracted from the line AB.

$\times$ denotes *into*, or *by*. The sign of *multiplication*, signifying that the numbers between which it is placed are to be multiplied together. Thus, 8×6 denotes that 8 is to be multiplied by 6. In geometrical lines also, $AB \times CD$ signifies that the line AB is to be multiplied by the line CD.

$\div$ denotes *divide by*. The sign of *division*, signifying that the former of the two numbers between which it is placed is to be divided by the latter. Thus, $8 \div 4$ denotes that 8 is to be divided by 4. This is also expressed by placing the dividend above a line, and the divisor below it. Thus, $\frac{8}{4}$ denotes that 8 is to be divided by 4. In geometrical lines also, $AB \div CD$ signifies that the line AB is to be divided by the line CD, or thus, $\frac{AB}{CD}$.

: is to
:: as.
: is to.

denote *proportionals*, signifying that the numbers between which they are placed are *proportional*. Thus, $2 : 4 :: 8 : 16$ denotes that the number 2 bears the same proportion to 4 as 8 does to 16, and is usually read, 2 is to 4 as 8 is to 16.

$=$ denotes *equal to*. The sign of *equality*, signifying that the numbers between which it is placed are equal to each other. Thus $2 + 2 = 4$ poles $= 22$ yards $=$ 1 chain $= 100$ links.

——— denotes a *vinculum*, and ( ) denotes a *parenthesis*. These signs are made use of to connect two or more quantities together. Thus, $(\overline{7 + 4} - 5) \times 8 = \overline{11 - 5} \times 8 = 6 \times 8 = 48$. The *vinculum* over the 7 and 4 serves as a chain to link them together, and shows that they are to be added together before the number 5 is subtracted; and the *parenthesis* also shows that the numerals which it includes must be operated upon, and the result multiplied by the number 8.

^{2} This sign placed above a quantity signifying that the quantity is to be *squared*. Thus, $\overline{5 + 3}|^2 = 8^2 = 8 \times 8 = 64$.

^{3} This sign placed above a quantity signifying that the quantity is to be *cubed*. Thus $\overline{(9+3) - 8}|^3 = \overline{12 - 8}|^3 = 4^3 = 4 \times 4 \times 4 = 64$.

$\sqrt{}$ denotes a *radical sign*, signifying that the quantity before which it is placed is to have its *square root extracted*. Thus,

$$\sqrt{\overline{(8+6)-(4\times 2)}|^2 + \overline{(\overline{12\times 9}\div 3)-(8+\overline{9-2})} + \overline{(\overline{7+5}\times 12)\div(3\times \overline{4\div 2})}}$$
$$= \sqrt{\overline{14-8}|^2 + \overline{(108\div 3)-(8+7)} + \overline{(12\times 12)\div(3\times 2)}} =$$
$$\sqrt{6^2 + (36-15) + (144\div 6)} = \sqrt{36+21+24} = \sqrt{81} = 9.$$

$\sqrt[3]{}$ denotes a *radical sign*, signifying that the quantity before which it is placed is to have its *cube root extracted*. Thus, $\sqrt[3]{(6\times 4\times 3)-8} = \sqrt[3]{(24\times 3)-8} = \sqrt[3]{72-8} = \sqrt[3]{64} = 4$.

∴ denotes *therefore*.

⊥ denotes a *perpendicular*.

∠ denotes an *angle*.

△ denotes a *triangle*.

′ denotes a *repeater*, when placed over a figure. Thus, ·819444, &c. is expressed by ·8194′.

″ denotes a *circulate* when placed over its first and last figure. Thus, ·769230769230, &c. is expressed by ·7′69230′

A KEY

TO

MR. KEITH'S COMPLETE MEASURER.

MENSURATION OF SUPERFICES.

PROBLEM I.

To find the Area of a Square.

Ex. 2. $AB^2 = 35{\cdot}25^2 = 1242{\cdot}5625$ square chains, and $\frac{1242{\cdot}5625}{10} \times 4 \times 40 =$ 124 acres, 1 rood, 1 perch, the area of the square ABCD = *Answer.*

Ex. 3. $AB^2 = \overline{5 \text{ ft. } 9 \text{ in.}}|^2 =$ 33 feet, 0 inches, 9 seconds, the area of the square ABCD = *Answer.*

Ex. 4. $AB^2 = 3725^2 = 13875625$ square links, and $\frac{13875625}{100000} \times 4 \times 40 =$ 138 acres, 3 roods, 1 perch, the area of the square ABCD = *Answer.*

PROBLEM II.

To find the Area of a Rectangle.

Ex. 2. AB × BC = 14 ft. 6 in. × 4 ft. 9 in. = 68 feet, 10 inches, 6 seconds, the area of the rectangle ABCD = *Answer.*

Ex. 3. AB × BC = 1375 × 950 = 1306250 square links, and $\frac{1306250}{100000} \times 4 \times 40 =$ 13 acres, 0 roods, 10 perches, the area of the rectangle ABCD = *Answer.*

Ex. 4. AB × BC = 2 ft. 10 in. 6 sec. × 9 in. = 2 ft. 1 in. 10 sec. 6 thirds, the area of the rectangle ABCD = *Answer.*

PROBLEM III.

To find the Area of a Rhombus, or of a Rhomboides.

Ex. 2. AB × AE = 6·2 × 5·45 = 33·79 square chains, and $\frac{33{\cdot}79}{10} \times 4 \times 40 =$ 3 acres, 1 rood, 20·64 perches, the area of the rhombus ABCD = *Answer.*

Ex. 3. AB × AE = 12 ft. 6 in. × 9 ft. 7 in. = 119 ft. 9 in. 6 sec. the area of the rhombus ABCD = *Answer.*

Ex. 4. AB × AE = 725 × 635 = 460375 square links, and $\frac{460375}{100000} \times 4 \times 40 =$ 4 acres, 2 roods, 16·6 perches, the area of the rhombus ABCD = *Answer.*

Ex. 5. AB × AE = 4784 × 1908 = 9127872 square links, and $\frac{9127872}{100000} \times 4 \times 40 =$ 91 acres, 1 rood, 4·5952 perches, the area of the rhomboides ABCD = *Answer.*

Ex. 6. AB × AE = 12 ft. 6 in. × 5 ft. 6 in. = 68 feet, 9 inches, the area of the rhomboides ABCD = *Answer.*

Ex. 7. AB × AE = 37 ft. × 5 ft. 3 in. = $\frac{37}{3}$ yards × $\frac{63}{36}$ yards = $\frac{2331}{108} = \frac{259}{12} = 21\frac{7}{12}$ square yards, the area of the rhomboides ABCD = *Answer.*

PROBLEM IV.

To find the Area of a Triangle.

Ex. 2. $\frac{1}{2}(28{\cdot}2 \times 18{\cdot}4) = 259.44$ square yards, the area of the triangle = *Answer.*

Ex. 3. $\frac{1}{2}(1236 \times 731) = 451758$ square links, and $\frac{451758}{100000} \times 4 \times 40 =$ 4 acres, 2 roods, 2·8128 perches, the area of the triangle = *Answer.*

Ex. 4. ½(18 ft. 4 in. × 11 ft. 10 in.) = 108 feet, 5 inches, 8 seconds, the area of the triangle = *Answer.*

PROBLEM V.

To find the Area of a Triangle, having the three Sides given.

Ex. 2. $\frac{1}{2}(AC + BC + AB) = \frac{1}{2}(50 + 40 + 30) =$ 60, the half sum of the sides.

$$\left.\begin{array}{l} 60 - 50 = 10 \\ 60 - 40 = 20 \\ 60 - 30 = 30 \end{array}\right\} \text{the three remainders.}$$

Then $\sqrt{60 \times 10 \times 20 \times 30} = \sqrt{360000} = 600$, the area of the triangle ABC = *Answer.*

Ex. 3. $\frac{1}{2}(BC + AB + AC) = \frac{1}{2}(41 + 29 + 56) =$ 63 yards, the half sum of the sides.

$$\left.\begin{array}{l} 63 - 41 = 22 \\ 63 - 29 = 34 \\ 63 - 56 = 7 \end{array}\right\}\text{the three remainders.}$$

Then $\sqrt{63 \times 22 \times 34 \times 7} = \sqrt{329868} = 574\cdot34$ square yards, the area of the triangle ABC = *Answer.*

Ex. 4. $\frac{1}{2}(BC + AC + AB) = \frac{1}{2}(4900 + 5025 + 2569) = 6247$ links, the half sum of the sides.

$$\left.\begin{array}{l} 6247 - 4900 = 1347 \\ 6247 - 5025 = 1222 \\ 6247 - 2569 = 3678 \end{array}\right\}\text{the three remainders.}$$

Then $\sqrt{6247 \times 1347 \times 1222 \times 3678} = \sqrt{37820044235844} = 6149800\cdot341136$ square links, and $\frac{6149800\cdot341136}{100000} \times 4 \times 40 =$ 61 acres, 1 rood, 39·6805458176 perches, the area of the triangle ABC = *Answer.*

Ex. 5. $\frac{1}{2}(AB + BC + AC) = \frac{1}{2}(380 + 420 + 765) = 782\cdot5$ yards, the half sum of the sides.

$$\left.\begin{array}{l} 782\cdot5 - 380 = 402\cdot5 \\ 782\cdot5 - 420 = 362.5 \\ 782\cdot5 - 765 = 17\cdot5 \end{array}\right\}\text{the three remainders.}$$

Then $\sqrt{782\cdot5 \times 402\cdot5 \times 362\cdot5 \times 17\cdot5} = \sqrt{1998003710\cdot9375} = 44699\cdot0347$ square yards, and $\frac{44699\cdot0347}{4840} = 9\cdot235337$ acres, the area of the triangle ABC = 1*st. Answer.*

Hence, as 1 sq. ac. : 9·235337 sq. ac. :: *55s.* : *25l.* *7s.* 11¼d. ·28968, the annual rent = 2*nd. Answer.*

PROBLEM VI.

Any two Sides of a right-angled Triangle being given, to find the third Side.

Ex. 3. By Rule 1. *$\sqrt{AB^2 + BC^2} = \sqrt{24^2 + 18^2} = \sqrt{900} = 30$, the hypothenuse AC = *Answer.*

Ex. 4. Here BC = 42·426 feet = 14.142 yards.

Then, by Rule 1. *$\sqrt{BC^2 + AB^2} = \sqrt{14{\cdot}142^2 + 23^2} = \sqrt{728{\cdot}996164} = 26{\cdot}999$ yards, or 27 yards nearly, the length of the chord or hypothenuse AC = *Answer.*

Ex. 5. By Rule 2. *$\sqrt{(AC + BC) \times (AC - BC)} = \sqrt{(30 + 18) \times (30 - 18)} = \sqrt{48 \times 12} = \sqrt{576} = 24$, the base AB = *Answer.*

Or thus:

*$\sqrt{AC^2 - BC^2} = \sqrt{30^2 - 18^2} = \sqrt{576} = 24$, the base AB = *Answer,* as before.

Ex. 6. *(See Fig. 1. of this Key.)* *$\sqrt{EG^2 - BG^2} = \sqrt{50^2 - 30^2} = \sqrt{1600} = 40$ feet = EB.

Again,*$\sqrt{EC^2 - AC^2} = \sqrt{50^2 - 40^2} = \sqrt{900} = 30$ feet = AE.

Then AE + EB = 30 + 40 = 70 feet = $23\frac{1}{3}$ yards, the breadth of the street AB = *Answer.*

Ex. 7. By Rule 2. *$\sqrt{(AC + BC) \times (AC - BC)} = \sqrt{(380 + 128) \times (380 - 128)} = \sqrt{508 \times 252} = \sqrt{128016} = 357{\cdot}79$ feet, the base AB = *Answer.*

Or thus:

*$\sqrt{AC^2 - BC^2} = \sqrt{380^2 - 128^2} = \sqrt{128016} = 357{\cdot}79$ feet, the base AB = *Answer,* as before.

---

* Euclid, 47 p. i. b.

Ex. 8. *(See Fig. 2. of this Key)* [*]$\sqrt{AC^2 - AB^2} = \sqrt{50^2 - 10^2} = \sqrt{2400} = 48\cdot9897$ feet $=$ BC, the height of the wall $=$ 1*st. Answer.*

Hence $BC - CE = 48\cdot9897 - 17\cdot9897 = 31$ feet $=$ BE, the height to the second-floor window.

Then[*]$\sqrt{BE^2 + DB^2} = \sqrt{31^2 + 6^2} = \sqrt{997} = 31\cdot5753$ feet $=$ DE, the length of a ladder to reach the second-floor window $=$ *2nd. Answer.*

PROBLEM VII.

To find the Area of a Trapezium.

Ex. 2. $\frac{1}{2}(DE + BF) \times AC = \frac{1}{2}(702 + 712) \times 1660 = 1173620$ square links, and $\frac{1173620}{100000} \times 4 \times 40$ $=$ 11 acres, 2 roods, 37·792 perches, the area of the field ABCD $=$ *Answer.*

Ex. 3. $\frac{1}{2}(BF + DE) \times AC = \frac{1}{2}(19\cdot75\text{ ft.} + 8\cdot75\text{ ft.}) \times 34\cdot75$ feet $=$ 495 feet, 2 inches, 3 seconds, the area of the trapezium ABCD $=$ *Answer.*

Ex. 4. *(See Fig. 3. of this Key.)* $\frac{1}{2}(AB + AD + BD) = \frac{1}{2}(3755 + 4105 + 4835) = 6347\cdot5$ links, the half sum of the sides.

$$\left.\begin{array}{l} 6347\cdot5 - 3755 = 2592\cdot5 \\ 6347\cdot5 - 4105 = 2242\cdot5 \\ 6347\cdot5 - 4835 = 1512\cdot5 \end{array}\right\}\text{the three remainders.}$$

Then $\sqrt{6347\cdot5 \times 2592\cdot5 \times 2242\cdot5 \times 1512\cdot5} = \sqrt{55814791873242\cdot1875} = 7470929\cdot78$ square links, the area of the north-west $\triangle$ ABD.

Again, $\frac{1}{2}(BC + CD + BD) = \frac{1}{2}(3575 + 2740 + 4835) = 5575$ links, the half sum of the sides.

$$\left.\begin{array}{l} 5575 - 3575 = 2000 \\ 5575 - 2740 = 2835 \\ 5575 - 4835 = 740 \end{array}\right\}\text{the three remainders}$$

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[*] Euclid, 47 p. i. b.

Then $\sqrt{5575 \times 2000 \times 2835 \times 740} = \sqrt{23391585000000} = 4836484$ square links, the area of the south-east $\triangle$ BDC.

$$\begin{array}{rl} \text{Hence the area of the } \triangle \text{ ABD} = & 7470929\cdot78 \\ \text{and the area of the } \triangle \text{ BDC} = & \underline{4836484} \\ \text{the sum} = & \underline{12307413\cdot78} \text{ sq. links,} \end{array}$$

and $\frac{12307413\cdot78}{100000} \times 4 \times 40 = 123$ acres, 0 roods, 11·862048 perches, the area of the four-sided field ABCD = *Answer.*

Ex. 5. $\frac{1}{2}(AB + BC + AC) = \frac{1}{2}(15 + 13 + 16) = 22$, the half sum of the sides.

$$\left.\begin{array}{l} 22 - 15 = 7 \\ 22 - 13 = 9 \\ 22 - 16 = 6 \end{array}\right\} \text{the three remainders.}$$

Then $\sqrt{22 \times 7 \times 9 \times 6} = \sqrt{8316} = 91\cdot1921$, the area of the $\triangle$ ACB.

Again, $\frac{1}{2}(CD + AD + AC) = \frac{1}{2}(14 + 12 + 16) = 21$, the half sum of the sides.

$$\left.\begin{array}{l} 21 - 14 = 7 \\ 21 - 12 = 9 \\ 21 - 16 = 5 \end{array}\right\} \text{the three remainders.}$$

Then $\sqrt{21 \times 7 \times 9 \times 5} = \sqrt{6615} = 81\cdot3326$, the area of the $\triangle$ ACD.

$$\begin{array}{rl} \text{Hence the area of the } \triangle \text{ ACB} = & 91\cdot1921 \\ \text{and the area of the } \triangle \text{ ACD} = & \underline{81\cdot3326} \\ \text{the sum} = & \underline{172\cdot5247} \text{ sq. links,} \end{array}$$

the area of the trapezium ABCD = *Answer.*

Ex. 6. [*] $\sqrt{AD^2 - AE^2} = \sqrt{220^2 - 100^2} = \sqrt{38400} = 195\cdot9591$ yards, the $\perp$ = DE.

Again, [*] $\sqrt{BC^2 - CF^2} = \sqrt{265^2 - 70^2} = \sqrt{65325} = 255\cdot5875$ yards, the $\perp$ = BF.

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[*] Euclid, 47 p. i. b.

Then $\frac{1}{2}(\mathrm{DE} + \mathrm{BF}) \times \mathrm{AC} = \frac{1}{2}(195{\cdot}9591 + 255{\cdot}5875) \times 378 = 85342{\cdot}3074$ square yards, and $\frac{85342{\cdot}3074}{4840} \times 4 \times 40 =$ 17 acres, 2 roods, 21·232 perches, the area of the trapezium ABCD = *Answer.*

PROBLEM VIII.

To find the Area of a Trapezoid.

Ex. 2. $\frac{1}{2}(\mathrm{DC} + \mathrm{AB}) \times \mathrm{CI} = \frac{1}{2}(750 + 1225) \times 1540 = 1520750$ square links, and $\frac{1520750}{100000} \times 4 \times 40 =$ 15 acres, 0 roods, 33·2 perches, the area of the trapezoid ABCD = *Answer.*

Ex. 3. $\frac{1}{2}(\mathrm{DC} + \mathrm{AB}) \times \mathrm{CI} = \frac{1}{2}(12\frac{1}{2} + 18\frac{1}{3}) \times 7\frac{3}{4} = 119\frac{23}{48}$ square feet = 119 feet, 5 inches, 9 seconds, the area of the trapezoid ABCD = *Answer.*

Ex. 4. $\frac{1}{2}(\mathrm{AB} + \mathrm{DC}) \times \mathrm{CI} = \frac{1}{2}(6340 + 4380) \times 121 = 648560$ square yards, and $\frac{648560}{4840} = 134$ sq. ac. the area of the field ABCD.

Then, as 134 sq. ac. : 1 sq. ac. :: 207*l.* 14*s.* : 1*l.* 11*s.* the yearly rent per acre = *Answer.*

PROBLEM IX.

To find the Area of an irregular Polygon.

Ex. 2. *(See Fig. 4. of this Key.)* $\frac{1}{2}(\mathrm{AB} + \mathrm{BC} + \mathrm{CA}) = \frac{1}{2}(2735 + 3115 + 3800) = 4825$ links, the half sum of the sides.

$$\left.\begin{array}{l}4825 - 2735 = 2090\\ 4825 - 3115 = 1710\\ 4825 - 3800 = 1025\end{array}\right\}\text{the three remainders.}$$

Then $\sqrt{4825 \times 2090 \times 1710 \times 1025} = \sqrt{17675169187500} = 4204184$ square links, the area of the △ ABC.

Again, $\frac{1}{2}(AC + AE + EC) = \frac{1}{2}(3800 + 2220 + 4010) = 5015$ links, the half sum of the sides.

$$\left.\begin{array}{l}5015 - 3800 = 1215\\ 5015 - 2220 = 2795\\ 5015 - 4010 = 1005\end{array}\right\}\text{the three remainders.}$$

Then $\sqrt{5015 \times 1215 \times 2795 \times 1005} = \sqrt{17115716694375} = 4137114$ square links, the area of the △ ACE.

Again, $\frac{1}{2}(CD + DE + EC) = \frac{1}{2}(2370 + 2925 + 4010) = 4652{\cdot}5$ links, the half sum of the sides.

$$\left.\begin{array}{l}4652{\cdot}5 - 2370 = 2282{\cdot}5\\ 4652{\cdot}5 - 2925 = 1727{\cdot}5\\ 4652{\cdot}5 - 4010 = 642{\cdot}5\end{array}\right\}\text{the three remainders.}$$

Then $\sqrt{4652{\cdot}5 \times 2282{\cdot}5 \times 1727{\cdot}5 \times 642{\cdot}5} = \sqrt{11786594866835{\cdot}9375} = 3433161{\cdot}06$ square links, the area of the △ CDE.

Hence the area of the △ ABC = 4204184
the area of the △ ACE = 4137114
and the area of the △ CDE = 3433161·06

the sum = 11774459.06 square links, and $\frac{11774459{\cdot}06}{100000} \times 4 \times 40 = 117$ acres, 2 roods, 39·134496 perches, the area of the five-sided field ABCDEA = *Answer.*

Ex. 3.

sq. links.

(See Rule, Prob. 4.) $\frac{1}{2}(AB \times BE) = \frac{1}{2}(440 \times 70) = 15400$ the area of the triangle ABE.

(See Rule, Prob. 8.) $\frac{1}{2}(\overline{BE + CF} \times BC) = \frac{1}{2}(\overline{70 + 320} \times 145) = 28275$ the area of the trapezoid EBCF.

(See Rule, Prob. 8.) $\frac{1}{2}(\overline{CF + DG} \times CD) = \frac{1}{2}(\overline{320 + 470} \times 405) = 159975$ the area of the trapezoid CDGF.

(See Rule, Prob. 4.) $\frac{1}{2}(DG \times DM) = \frac{1}{2}(470 \times 30) = 7050$ the area of the triangle DMG.

(See Rule, Prob. 4.) $\frac{1}{2}(HK \times AH) = \frac{1}{2}(350 \times 315) = 55125$ the area of the triangle AHK.

(See Rule, Prob. 8.) $\frac{1}{2}(\overline{HK + IL} \times HI) = \frac{1}{2}(\overline{350 + 50} \times 295) = 59000$ the area of the trapezoid HILK.

(See Rule, Prob. 4.) $\frac{1}{2}(IM \times IL) = \frac{1}{2}(410 \times 50) = 10250$ the area of the triangle IML.

the sum $= 335075$ square links, and

$\frac{335075}{100000} \times 4 \times 40 = 3$ acres, 1 rood, 16·12 perches, the area of the irregular polygon AEF GMLKA $=$ *Answer.*

PROBLEM X.

To find the Area of a regular Polygon.

Ex. 2. By Rule 1. $14·6 \times 6 = 87·6$, the perimeter, or sum of the sides.

Then $\frac{1}{2}(87·6 \times 12·64397) = 553·805886$, the area of the hexagon = *Answer*.

Ex. 3. By Rule 1. $9·941 \times 8 = 79·528$, the sum of the sides.

Then $\frac{1}{2}(79·528 \times 12) = 477·168$, the area of the octagon = *Answer*.

Ex. 4. By Rule 1. $19·38 \times 7 = 135·66$, the sum of the sides.

Then $\frac{1}{2}(135·66 \times 20·1215) = 1364·841345$, the area of the heptagon = *Answer*.

Ex. 5. By Rule 2. $20^2 \times 7·694209 = 3077·6836$, the area of the decagon = *Answer*.

Ex. 6. By Rule 1. $102 \times 12 = 1224$, the sum of the sides.

Then $\frac{1}{2}(1224 \times 190·3345908) = 116484·7695696$, the area of the duodecagon = *Answer*.

Ex. 7. By Rule 1. $40 \times 9 = 360$, the sum of the sides.

Then $\frac{1}{2}(360 \times 54·949548) = 9890·91864$, the area of the nonagon = *Answer*.

Ex. 8. By Rule 2. $20^2 \times 9·365640 = 3746·256$, the area of the undecagon = *Answer*.

Ex. 9. By Rule 1. $20 \times 3 = 60$, the sum of the sides.

Then $\frac{1}{2}(60 \times 5·773502) = 173·20506$, the area of the trigon = *Answer*.

Ex. 10. By Rule 1. $\frac{1}{2}(20 \times 4 \times 10) = 400$, the area of the tetragon = *Answer*.

PROBLEM XI.

Given the Diameter of a Circle to find the Circumference, or the Circumference to find the Diameter.

Ex. 3. By Rule 1. Case 3. 7970 × 3·1416 = 25038·552 miles, the circumference = *Answer*.

Ex. 4. By Rule I. Case 3. 50 × 3·1416 = 157·08 feet, the circumference = *Answer*.

Ex. 5. By Rule 2. Case 3. 12 × ·318309 = 3·819708, the diameter = *Answer*.

PROBLEM XII.

To find the Length of any Arc of a Circle.

Ex. 3. By Rule 2. $\frac{1}{3}(8\,AD - AB) = \frac{1}{3}(\overline{3{\cdot}043836 \times 8} - 6) = 6{\cdot}116896$, the length of the arc ADB = *Answer*.

Ex. 4. By Rule 1. OA × ADB° × ·01745329 = 9 × 38°·9424412 × ·01745329 = 6·117063476143932, the length of the arc ADB = *Answer*.

Ex. 5. By Rule 2. $\frac{1}{3}(8\,AD - AB) = \frac{1}{3}(\overline{30{\cdot}6 \times 8 - 50{\cdot}8}) = 64\frac{2}{3}$ inches, the length of the arc ADB = *Answer*.

Ex. 6. *$\sqrt{AC^2 + CD^2} = \sqrt{20^2 + 15^2} = \sqrt{625} = 25 = AD$, the chord of half the arc ADB.

Then, by Rule 2. $\frac{1}{3}(8\,AD - AB) = \frac{1}{2}(\overline{25 \times 8} - 40) = 53\frac{1}{3}$ the length of the arc ADB = *Answer*.

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* Euclid, 47 p. I. B.

Ex. 7. By Rule 1. $OA \times ADB \times {\cdot}01745329 = 11{\cdot}3 \times 52{\cdot}25^\circ \times {\cdot}01745329 = 10{\cdot}30485874825$, the length of the arc ADB = *Answer.*

Ex. 8 By Note 8. †$AC^2 = EC \times CD$, $\therefore EC = AC^2 \div CD = 18^2 \div 6 = 54$; $\therefore ED = EC + CD = 54 + 6 = 60$, the diameter = *Answer.*

Ex. 9. By Note 8. †$AC^2 = EC \times CD$, $\therefore EC = AC^2 \div CD = 9^2 \div 4 = 20\frac{1}{4}$; $\therefore ED = EC + CD = 20\frac{1}{4} + 4 = 24\frac{1}{4}$, the diameter = *Answer.*

Ex. 10. *$\sqrt{OA^2 - CA^2} = \sqrt{10^2 - 6^2} = \sqrt{64} = 8 = OC$; hence $OD - OC = 10 - 8 = 2 = CD$, the versed sine. Now,*$\sqrt{CA^2 + CD^2} = \sqrt{6^2 + 2^2} = \sqrt{40} = 6{\cdot}32455532 = AD$, the chord of half the arc ADB. Then, by Rule 2, $\frac{1}{3}(8AD - AB) = \frac{1}{3}(\overline{6{\cdot}32455532 \times 8 - 12}) = 12.86548085$, the length of the arc ADB = *Answer.*

Or, find the length of the arc thus:

By Note to Rule 2. $$\left(5ED\sqrt{\frac{5CD}{5ED - 3CD}} + 4\sqrt{ED \times CD}\right) \times \tfrac{2}{9} = \left(5 \times 20\sqrt{\frac{5 \times 2}{(5 \times 20) - (3 \times 2)}} + 4\sqrt{20 \times 2}\right) \times \tfrac{2}{9} = \left(100\sqrt{\frac{10}{94}} + 4\sqrt{40}\right) \times \tfrac{2}{9} = \tfrac{8}{9}\left(25\sqrt{\frac{5}{47}} + 2\sqrt{10}\right) = \tfrac{8}{9}\left(\frac{25}{47}\sqrt{235} + 2\sqrt{10}\right) = \tfrac{8}{9}(8{\cdot}1541009 + 6{\cdot}3245554) = 12{\cdot}8699167,$$ the length of the arc ADB = *Answer*, as before nearly.

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† Euclid, 35. p, iii. b. * Euclid, 47. p, i. b.

PROBLEM XIII.

To find the Area of a Circle.

Ex. 2. By Rule 1. $\frac{1}{4}(3{\cdot}1416 \times 1) = {\cdot}7854$, the area of the circle = *Answer.*

Ex. 3. By Rule 2. $22{\cdot}6^2 \times {\cdot}7854 = 401{\cdot}150904$, the area of the circle = *Answer.*

Ex. 4. By Rule 3. As $452 : 355 :: 22{\cdot}6^2 : 401{\cdot}15$, the area of the circle = *Answer.*

Ex. 5. By Rule 4. As $14 : 11 :: 22.6^2 : 401{\cdot}31$, the area of the circle = *Answer.*

Ex. 6. By Rule 5. $71^2 \times {\cdot}07958 = 401{\cdot}16278$, the area of the circle = *Answer.*

Ex. 7. By Rule 6. As $88 : 7 :: 71^2 : 400{\cdot}9886$, the area of the circle = *Answer.*

Ex. 8. By Rule 7. As $1420 : 113 :: 71^2 : 401{\cdot}15$, the area of the circle = *Answer.*

Ex. 9. By Rule 2. $12^2 \times {\cdot}7854 = 113{\cdot}0976$, the area of the circle = *Answer.*

Ex. 10. By Rule 1. $\frac{1}{4}(12 \times 37{\cdot}6992) = 113{\cdot}0976$, the area of the circle = *Answer.*

Ex. 11. By Rule 5. $37{\cdot}6992^2 \times {\cdot}07958 = 113{\cdot}1014579853312$, the area of the circle = *Answer.*

Ex. 12. $200 \div 2 = 100$ poles, the circumference of the bowling-green.

Then, by Rule 5, $100^2 \times {\cdot}07958 = 795{\cdot}8$ square poles, or perches, and $\frac{795{\cdot}8}{160} \times 4 \times 40 = 4$ acres, 3 roods, 35.8 perches, the area of the bowling-green = *Answer.*

Ex. 13. $27{\cdot}75 \times 2 = 55{\cdot}5$ yards, the diameter. Then, by Rule 2, $55{\cdot}5^2 \times {\cdot}7854 = 2419{\cdot}22835$ square yards, the quantity of ground the fish-pond took up = *Answer*.

Ex. 14. By Rule 1, and Note. $\frac{1}{2}(47{\cdot}124 \times 15) = 353{\cdot}43$, the area of the semicircle = *Answer*.

Ex. 15. By Rule 2, and Note. $\frac{1}{2}(200^2 \times {\cdot}7854) = 15708$, the area of the semicircle = *Answer*.

Ex. 16. By Rule 1, and Note. $\frac{1}{2}\left(\frac{157{\cdot}08 \times 50}{4}\right) = 981{\cdot}75$, the area of the semicircle = *Answer*.

Ex. 17. $23{\cdot}562 \times 2 = 47{\cdot}124$, half the circumference; and, by the question, half the diameter = 15. Then, by Rule 1, and Note, $\frac{1}{4}(47{\cdot}124 \times 15) = 176.715$, the area of the quadrant = *Answer*.

Ex. 18. By Rule 2, and Note. $\frac{1}{4}(200^2 \times {\cdot}7854) = 7854$, the area of the quadrant = *Answer*.

PROBLEM XIV.

Given the Diameter, Circumference, or Area of a Circle; to find the Side of a Square equal in Area to the Circle, and the Side of a Square inscribed in the Circle: or, having the Side of a Square given, to find the Diameter of its circumscribing Circle, and also of a Circle equal in Area, &c.

Ex. 1. By Rule 1. $22{\cdot}6 \times {\cdot}8862269 = 20{\cdot}02872794$, the side of the square equal in area to the circle = *Ans.*

Ex. 2. By Rule 2. $40 \times {\cdot}2820948 = 11{\cdot}283792$, the side of the square equal in area to the circle = *Ans.*

Ex. 3. By Rule 3. $50 \times {\cdot}7071068 = 35{\cdot}35534$, the side of the square inscribed in the circle = *Answer*.

Then, by Rule 2. First, $\sqrt{AF^2+FG^2} = \sqrt{10{\cdot}25^2+6{\cdot}119^2} = \sqrt{142{\cdot}504661} = 11{\cdot}9375 =$ AG, the chord of half the arc AGB. Secondly, $\sqrt{AB^2+FG^2} = \sqrt{20{\cdot}5^2+6{\cdot}119^2} = \sqrt{457{\cdot}692161} = 21{\cdot}3937$. Then $(\overline{21{\cdot}3937 \times 2} + AG) \times FG \times \frac{4}{15} = (\overline{21{\cdot}3937 \times 2} + 11{\cdot}9375) \times 6{\cdot}119 \times \frac{4}{15} = 89{\cdot}2964434$, the area of the less segment ABGA.

Hence the area of the whole circle AGBEDA } = 425·983305,
and the area of the less segment ABGA } = 89·2964434;
the difference = 336·6868615, the area of the greater segment ADEBFA = *Answer*.

Ex. 5. By Rule 1. [Note 8, Prob. XII.] $(AF^2 \div FG) + FG = (9{\cdot}45^2 \div 2{\cdot}4) + 2{\cdot}4 = 39{\cdot}609375$, the diameter DG, which ÷ by 2 = 19·8046875, the radius CG; ∴ $CG - FG = 19{\cdot}8046875 - 2{\cdot}4 = 17{\cdot}4046875 =$ CF, the ⊥ of the △ ACB. Now, *$\sqrt{AF^2 + FG^2} = \sqrt{9.45^2 + 2{\cdot}4^2} = \sqrt{95{\cdot}0625} = 9{\cdot}75 =$ AG, the chord of half the arc AGB. Then, by Rule, Prob. IV, $\frac{1}{2}(CF \times AB) = \frac{1}{2}(17{\cdot}4046875 \times 18{\cdot}9) = 164{\cdot}474296875$, the area of the △ ACB. Then, by Rule 2, Prob. XII, $\frac{1}{3}(8AG - AB) = \frac{1}{3}(\overline{9{\cdot}75 \times 8} - 18{\cdot}9) = 19{\cdot}7$, the length of the arc AGB, which ÷ by 2 = 9·85, the length of the arc AG, or = ½ AGB. Then, by Rule 1, Prob. XV, the arc $AG \times CG = 9{\cdot}85 \times 19{\cdot}8046875 = 195{\cdot}076171875$, the area of the sector ACBGA.

Hence the area of the sector ACBGA } = 195·076171875,
and the area of the triangle ACB } = 164·474296875;
the difference = 30·601875000, the area of the segment ABGA = *Answer*.

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* Euclid, 47. p, i. b.

By Rule 2. First, $\sqrt{AF^2 + FG^2} = \sqrt{9·45^2 + 2·4^2} = \sqrt{95·0625} = 9·75 = AG$, the chord of half the arc AGB. Secondly, $\sqrt{AB^2 + FG^2} = \sqrt{18·9^2 + 2·4^2} = \sqrt{362·97} = 19·05177$. Then $(\overline{19·05177 \times 2} + AG) \times FG \times \frac{4}{15} = (\overline{19·05177 \times 2} + 9·75) \times 2·4 \times \frac{4}{15} = 30·6262656$, the area of the segment ABGA = *Answer*.

Note,—Rule 3. $\sqrt{AF^2 + FG^2} = \sqrt{9·45^2 + 2·4^2} = \sqrt{95·0625} = 9·75 = AG$, the chord of half the arc AGB. Then $(\frac{4}{3}AG + AB) \times \frac{2}{5}FG = (\overline{9·75 \times \frac{4}{3}} + 18·9) \times (2·4 \times \frac{2}{5}) = 30·624$, the area of the segment ABGA = *Answer*.

Note,—Rule 4. $\frac{2}{3}(AB \times FG) + (FG^3 \div 2AB) = (18·9 \times 2·4 \times \frac{2}{3}) + (2·4^3 \div \overline{18·9 \times 2}) = 30·605714$, the area of the segment ABGA = *Answer*.

Ex. 6. By Rule 1. [Note 8, Prob. XII.] $(AF^2 \div FG) + FG = (11·5^2 \div 3·5) + 3·5 = 41·28$, the diameter DG, which ÷ by 2 = 20·64, the radius CG; ∴ $CG - FG = 20·64 - 3·5 = 17·14 = CF$, the ⊥ of the △ ACB. Now, *$\sqrt{AF^2 + FG^2} = \sqrt{11·5^2 + 3.5^2} = \sqrt{144·50} = 12·0208 = AG$, the chord of half the arc AGB. Then, by Rule, Prob. IV, $\frac{1}{2}(CF \times AB) = \frac{1}{2}(17·14 \times 23) = 197·11$, the area of the △ ACB. Then, by Rule 2, Prob. XII, $\frac{1}{3}(8AG - AB) = \frac{1}{3}(\overline{12·0208 \times 8} - 23) = 24·3888$, the length of the arc AGB, which ÷ by 2 = 12·1944, the length of the arc AG. Then, by Rule 1, Prob. XV, the arc $AG \times CG = 12·1944, \times 20·64 = 251·692416$, the area of the sector ACBGA.

Hence the area of the sector ACBGA = 251·692416,
and the area of the triangle ACB = 197·110000;
the difference = 54·582416,

the area of the segment ABGA = *Answer*.

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* Euclid, 47. p, i. b.

By Rule 2. First, $\sqrt{AF^2 + FG^2} = \sqrt{11{\cdot}5^2 + 3{\cdot}5^2} = \sqrt{144{\cdot}50} = 12{\cdot}02081 = AG$, the chord of half the arc AGB. Secondly, $\sqrt{AB^2 + FG^2} = \sqrt{23^2 + 3{\cdot}5^2} = \sqrt{541{\cdot}25} = 23{\cdot}26478$. Then, $(\overline{23{\cdot}26478 \times 2} + AG) \times FG \times \frac{4}{15} = (\overline{23{\cdot}26478 \times 2} + 12{\cdot}02081) \times 3{\cdot}5 \times \frac{4}{15} = 54{\cdot}647012$, the area of the segment ABGA = *Answer*.

Ex. 7. By Note 8, Prob. XII. $AF^2 \div FD = 6^2 \div 18 = 2 = FG$, the versed sine of the smaller segment ABGA; $\therefore$ $FG + FD = 2 + 18 = 20$, the diameter DG. Then, by Rule 2, Prob. XIII, $DG^2 \times {\cdot}7854 = 20^2 \times {\cdot}7854 = 314{\cdot}16$, the area of the whole circle ADE BGA. Then, by Rule 2. First, $\sqrt{AF^2 + FG^2} = \sqrt{6^2 + 2^2} = \sqrt{40} = 6{\cdot}32455 = AG$, the chord of half the arc AGB. Secondly, $\sqrt{AB^2 + FG^2} = \sqrt{12^2 + 2^2} = \sqrt{148} = 12{\cdot}165525$. Then, $(\overline{12{\cdot}165525 \times 2} + AG) \times FG \times \frac{4}{15} = (\overline{12{\cdot}165525 \times 2} + 6{\cdot}32455) \times 2 \times \frac{4}{15} = 16{\cdot}349653$, the area of the smaller segment ABGA.

Hence the area of the whole circle ADEBGA } = 314·160000,
and the area of the smaller segment ABGA } = 16·349653;

the difference = 297·810346, the area of the greater segment ADEBFA = *Answer*.

PROBLEM XVII.

To find the Area of the Segment of a Circle by the Table given at the end of the Treatise.

Ex. 4. By Note 8, Prob. XII. $(6^2 \div 18) + 18 = 20$, the diameter. Then, by Rule and Note, [Exam. 2.], 18 ÷ by 20 = ·900; hence 1 — ·900 = ·100, the tabular height, against which stands the *Area Seg.* ·040875;

then ·7854 — ·040875 = ·744525, the correct tabular *Area Seg.*; hence ·744525 × 20^2 = 297·81, the area of the segment = *Answer.*

Ex. 5. By Note 8, Prob. XII. $(10^2 \div 5) + 5 = 25$, the diameter. Then, by Rule, [Exam. 1.], 5 ÷ by 25 = ·200, the tabular height, against which stands the *Area Seg.* ·111823; hence ·111823 × 25^2 = 69·889375, the area of the segment = *Answer.*

PROBLEM XVIII.

To find the Area of a Circular Zone, or the Space included by two parallel Chords, and the Arcs contained between them.

Ex. 2. By Note to this Problem.—*General Construction*, and *General Method of Solution.* $\frac{1}{2}(AB - CD) = \frac{1}{2}(96 - 60) = 18 = mB$; ∴ *$\sqrt{Dm^2 + mB^2} = \sqrt{26^2 + 18^2} = \sqrt{1000} = 31{\cdot}62277 = DB = AC$; and $AB - mB = 96 - 18 = 78 = Am$. Then, by Note 8, Prob. XII, $(Am \times mB) \div Dm = (78 \times 18) \div 26 = 54 = mE$, and $Dm + mE = 26 + 54 = 80 = DE$; ∴ *$\frac{1}{2}\sqrt{CD^2 + DE^2} = \frac{1}{2}\sqrt{60^2 + 80^2} = \frac{1}{2}\sqrt{3600 + 6400} = \frac{1}{2}\sqrt{10000} = 50$, the radius of the circle $= \frac{1}{2}CE = FH$. Now, by the Note, or (Euclid, 4. p, vi. b.), as $DB : Dm :: AE\ (= 2OF) : Am$; ∴ $(DB \times Am) \div 2Dm = (31{\cdot}62277 \times 78) \div (26 \times 2) = 47{\cdot}43415 = OF$; ∴ $FH - OF = 50 - 47{\cdot}43415 = 2{\cdot}56584 = OH$, the height of the segment AHC. Then, by Note, and Exam. 3, Prob. XVII, $OH \div CE = 2{\cdot}56584 \div 100 = {\cdot}0256584$, the tabular *Area Seg.* corresponding to which is ·0054380544; hence $\cdot0054380544 \times 100^2 = 54{\cdot}380544$, the area of the segment AHC. Then, by Rule, Prob. VIII, $\frac{1}{2}(\overline{AB + CD} \times Dm) = \frac{1}{2}(\overline{96 + 60} \times 26) = 2028$, the area of the trapezoid ABDCA.

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*Euclid, 47. p, i. b.

Then, by the Rule, $\mathrm{ABDCA} + 2\mathrm{AHC} = 2028 + \overline{54.380544 \times 2} = 2136{\cdot}761088$, the area of the zone $\mathrm{ABDCHA} =$ *Answer.*

Ex. 3. $\frac{1}{2}(\mathrm{AB} - \mathrm{CD}) = \frac{1}{2}(48 - 30) = 9 = m\mathrm{B}$. Now, $\mathrm{CA} = \mathrm{DB}$; hence* $\sqrt{\mathrm{DB}^2 - m\mathrm{B}^2} = \sqrt{15{\cdot}8114^2 - 9^2} = \sqrt{169{\cdot}00036996} = 13 = \mathrm{D}m$; $\therefore \mathrm{AB} - m\mathrm{B} = 48 - 9 = 39 = \mathrm{A}m$. Then, by Note 8, Prob. XII, $(\mathrm{A}m \times m\mathrm{B}) \div \mathrm{D}m = (39 \times 9) \div 13 = 27 = m\mathrm{E}$, and $\mathrm{D}m + m\mathrm{E} = 13 + 27 = 40 = \mathrm{DE}$; $\therefore$ * $\frac{1}{2}\sqrt{\mathrm{CD}^2 + \mathrm{DE}^2} = \frac{1}{2}\sqrt{30^2 + 40^2} = \frac{1}{2}\sqrt{900 + 1600} = \frac{1}{2}\sqrt{2500} = 25$, the radius of the circle $= \frac{1}{2}\mathrm{CE} = \mathrm{FH}$. Now, by Note to this Problem, $(\mathrm{DB} \times \mathrm{A}m) \div 2\mathrm{D}m = (15{\cdot}8114 \times 39) \div (13 \times 2) = 23{\cdot}7171 = \mathrm{OF}$; and $\mathrm{FH} - \mathrm{OF} = 25 - 23{\cdot}7171 = 1{\cdot}2829 = \mathrm{OH}$, the height of the segment AHC. Then, by Note, and Exam. 3, Prob. XVII, $\mathrm{OH} \div \mathrm{CE} = 1{\cdot}2829 \div 50 = {\cdot}025658$, the tabular *Area Seg.* corresponding to which is ·005437928; hence ${\cdot}005437928 \times 50^2 = 13{\cdot}59482$, the area of the segment AHC. Then, by Rule, Prob. VIII, $\frac{1}{2}(\overline{\mathrm{AB} + \mathrm{CD}} \times \mathrm{D}m) = \frac{1}{2}(\overline{48 + 30} \times 13) = 507$, the area of the trapezoid ABDCA. Then, by the Rule, $\mathrm{ABDCA} + 2\mathrm{AHC} = 507 + \overline{13{\cdot}59482 \times 2} = 534{\cdot}18964$, the area of the zone $\mathrm{ABDCHA} =$ *Answer.*

Ex. 4. *(See Fig. 6. of this Key).* By Rule 2, Prob. XVI. First, $\sqrt{\mathrm{AO}^2 + \mathrm{HO}^2} = \sqrt{9{\cdot}45^2 + 2{\cdot}4^2} = \sqrt{95{\cdot}0625} = 9{\cdot}75 = \mathrm{AH}$, the chord of half the arc AHC. Secondly, $\sqrt{\mathrm{AC}^2 + \mathrm{HO}^2} = \sqrt{18{\cdot}9^2 + 2{\cdot}4^2} = \sqrt{362{\cdot}97} = 19{\cdot}05177$. Then, $(\overline{19{\cdot}05177 \times 2} + \mathrm{AH}) \times \mathrm{HO} \times \frac{4}{15} = (\overline{19{\cdot}05177 \times 2} + 9{\cdot}75) \times 2{\cdot}4 \times \frac{4}{15} = 30{\cdot}6262656$, the area of the segment AHC. Now, half the chord $\mathrm{ED} = \mathrm{E}r = 11{\cdot}5$, and the height of the segment $\mathrm{EBSD} = r\mathrm{S} = 3{\cdot}5$. Then, by Rule 2, Prob. XVI. First, $\sqrt{\mathrm{E}r^2 + r\mathrm{S}^2} = \sqrt{11{\cdot}5^2 + 3{\cdot}5^2} = \sqrt{144{\cdot}5} = 12{\cdot}02081 = \mathrm{ES}$, the chord of half the arc DSBE. Secondly, $\sqrt{\mathrm{ED}^2 + r\mathrm{S}^2} = \sqrt{23^2 + 3{\cdot}5^2} = \sqrt{541{\cdot}25} =$

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* Euclid, 47. p, i. b.

23·26478. Then, $(\overline{23\cdot26478 \times 2} + \text{ES}) \times r\text{S} \times \frac{4}{15} = (\overline{23.26478 \times 2} + 12\cdot02081) \times 3\cdot5 \times \frac{4}{15} = 54\cdot647012$, the area of the segment EBSD. Now, the ⊥ let fall from D upon the diagonal CE = DF = 16·2; and that from A to the diagonal CE = An = 16. Then, by Rule, Prob. VII, $\frac{1}{2}(\overline{\text{DF} + \text{A}n} \times \text{CE}) = \frac{1}{2}(\overline{16\cdot2 + 16} \times 34) = 547\cdot4$, the area of the trapezium AEDCA.

Hence the area of the segment AHC = 30·6262656,
the area of the segment EBSD = 54·647012,
and the area of the trapezium AEDCA = 547·400000;

the sum = 632·6732776, the area of the whole compound figure AEBSDCHA = *Answer*.

PROBLEM XIX.

To find the area of a circular Ring, or of the Space included between two concentric Circles.

Ex. 2. (AB + CD) × (AB − CD) × ·7854 = (24 + 18) × (24 − 18) × ·7854 = 42 × 6 × .7854 = 197·9208 square feet, the area of the ring *mm* = *Answer*.

Ex. 3. (AB + CD) × (AB − CD) × ·7854 = (32 + 25) × (32 − 25) × ·7854 = 57 × 7 × ·7854 = 313·3746 square feet, the area of the ring *mm* = *Answer*.

Ex. 4. The longer diameter AB = 56 yards, and the shorter diameter CD = 52 yards; hence (AB + CD) × (AB − CD) × ·7854 = (56 + 52) × (56 − 52) × ·7854 = 108 × 4 × ·7854 = 339·2928 square yards, the area of the gravel walk *mm*. Then, as 1 sq. yd. : 339·2928 sq. yds. :: 2*s*. 6*d*. : £42. 8*s*. $2\frac{3}{4}$*d*.·136, the expense of the gravel = *Answer*.

Ex. 5. By the Note. [See Diagram]. $\frac{1}{2}(\overline{DE + BC} \times BD) = \frac{1}{2}(\overline{37 \text{ ft. } 6 \text{ in.} + 36 \text{ ft. } 4 \text{ in.}} \times 2\text{ft. } 6 \text{ in.})$ $=$ 92 feet, 3 inches, 6 seconds, the area of the circular part of the arc DECBD $=$ *Answer.*

PROBLEM XX.

To find the area of a Lune, or the Space included between the intersecting Arcs of two Eccentric Circles.

Ex. 2. *(See Fig. 7. of this Key.)* By Rule 2, Prob. XVI. First, $\sqrt{AE^2 + EC^2} = \sqrt{20^2 + 15^2} = \sqrt{625} = 25 = AC$, the chord of half the arc ACB. Secondly, $\sqrt{AB^2 + EC^2} = \sqrt{40^2 + 15^2} = \sqrt{1825} = 42{\cdot}720018$. Then, $(\overline{42{\cdot}720018 \times 2} + AC) \times EC \times \frac{4}{15} = (\overline{42{\cdot}720018 \times 2} + 25) \times 15 \times \frac{4}{15} = 441{\cdot}760144$, the area of the greater segment ABCA. Again, by Rule 2, Prob. XVI. First, $\sqrt{AE^2 + ED^2} = \sqrt{20^2 + 2^2} = \sqrt{404} = 20{\cdot}099751 = AD$, the chord of half the arc ADB. Secondly, $\sqrt{AB^2 + ED^2} = \sqrt{40^2 + 2^2} = \sqrt{1604} = 40{\cdot}049968$. Then, $(\overline{40{\cdot}049968 \times 2} + AD) \times ED \times \frac{4}{15} = (\overline{40{\cdot}049968 \times 2} + 20{\cdot}099751) \times 2 \times \frac{4}{15} = 53{\cdot}439833$, the area of the less segment ABDA.

Hence the area of the greater segment ABCA $= 441{\cdot}760144$,
and the area of the less segment ABDA $= 53{\cdot}439833$;

the difference $= 388{\cdot}320311$, the area of the lune ADBCA $=$ *Answer.*

Ex. 3. By Note 8, Prob. XII. $(AE^2 \div EC) + EC = (12^2 \div 9) + 9 = 25$, the diameter of the circle of which ACB is an arc. Again, by Note 8, Prob. XII,

$(AE^2 \div ED) + ED = (12^2 \div 4) + 4 = 40$, the diameter of the circle of which ADB is an arc. Then, by Rule, Prob. XVII. Exam. 1, $9 \div 25 = \cdot360$, the tabular *Area Seg.* corresponding to which is $\cdot254550$; hence $\cdot254550 \times 25^2 = 159\cdot09375$, the area of the greater segment ABCA. Again, by Rule, Prob. XVII. Exam. 1, $4 \div 40 = \cdot100$, the tabular *Area Seg.* corresponding to which is $\cdot040875$; hence $\cdot040875 \times 40^2 = 65\cdot4$, the area of the less segment ABDA.

$$\left.\begin{array}{l}\text{Then, the area of the}\\ \text{greater segment ABCA}\end{array}\right\} = 159\cdot09375,$$

$$\left.\begin{array}{l}\text{And the area of the less}\\ \text{segment ABDA}\end{array}\right\} = 65\cdot40000;$$

the difference $= 93\cdot69375$, the area of the lune ADBCA = *Answer*.

THE
MENSURATION OF THE CONIC SECTIONS.

1.—OF THE ELLIPSE.

PROBLEM XXI.*

The transverse and conjugate Diameters of an Ellipse being given, to find the Area.

Ex. 2. AB × CD × ·7854 = 50 × 40 × ·7854 = 1570·8, the area of the ellipse ACBDA = *Answer.*

Ex. 3. AB × CD × ·7854 = 70 × 50 × ·7854 = 2748·9, the area of the ellipse ACBDA = *Answer.*

Ex. 4. AB × CD × ·7854 = 24 × 18 × ·7854 = 339·2928, the area of the ellipse ACBDA = *Answer.*

PROBLEM XXII.

To find the Area of an Elliptical Ring, or the Space included between the Circumferences of two concentric and similar Ellipses.

Ex. 2. By Rule Prob. XXI. EF × GH × ·7854 = 840 × 612 × ·7854 = 403758·432 square links, and $\frac{403758\cdot432}{100000} \times 4 \times 40 =$ 4 acres, 0 roods, 6·0134912 perches, the area of the ellipse within the wall EGFHE = 1*st Ans.* Then $(\overline{AE+FB}\div 7\cdot92$ inches in a link$)+EF=(\overline{14\,\text{in.}+}$

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* See the Diagram to Prob. XXII.

$\overline{14\text{ in.}} \div 7{\cdot}92) + 840 = 843{\cdot}53535$ links, the outer transverse diameter AB, and $(\overline{CG + HD} \div 7{\cdot}92) + GH = (\overline{14\text{ in.} + 14\text{ in.}} \div 7{\cdot}92) + 612 = 615{\cdot}53535$ links, the outer conjugate diameter CD. Then, by the Rule, $AB \times CD = 843{\cdot}53535 \times 615{\cdot}53535 = 519225{\cdot}8268996225$ links the product of the two outer diameters, and $EF \times GH = 840 \times 612 = 514080$ links, the product of the two inner diameters; hence $\overline{519225{\cdot}8268996225 - 514080} \times {\cdot}7854 = 4041{\cdot}5324469635115$ square links, which ÷ by 20·6611 (sq. links in a sq. yard) = 195·6107103185 square yards, the contents of the ground on which the wall stands = *2nd Answer.*

Ex. 3. $AE + EF + FB = 5\frac{1}{2} + 30 + 5\frac{1}{2} = 41$ feet, the outer transverse diameter AB, and $CG + GH + HD = 5\frac{1}{2} + 24 + 5\frac{1}{2} = 35$ feet, the outer conjugate diameter CD. Then, by the Rule, $AB \times CD = 41 \times 35 = 1435$ feet, the product of the two outer diameters, and $EF \times GH = 30 \times 24 = 720$ feet, the product of the two inner diameters; hence $\overline{1435 - 720} \times {\cdot}7854 = 561{\cdot}561$ square feet, the area of the walk = 1*st Answer.*

Then 1 sq. yard : 561·561 sq. feet : : 4*s.* : £12. 9*s.* 6¾*d.* ·968 the expense of paving the elliptical walk = *2nd Answer.*

PROBLEM XXIII.

Given the Height of an Elliptical Segment, whose Base is parallel to either of the Axes of the Ellipse, and the two Axes to find the Area.

Ex. 2. (*See* Note 8, Prob. XII. *or Euclid,* 35 p. iii. b.) (*See Fig.* 8 *of this Key.*) By Rule 1 $(CD - PC) \times PC = (40 - 4) \times 4 = 144 = rP \times Pe$ or $= rP^2$ the square of half the chord of the *circular* segment *reCr*, whose height is $4 = PC$ and $2\sqrt{rP^2} = 2\sqrt{144} = 12 \times 2 = 24 = re$ the chord of the *circular* segment *reCr.*

Then, by Rule 2, Prob. XVI. First, $\sqrt{rP^2 + PC^2} = \sqrt{144 + 16} = \sqrt{160} = 12{\cdot}64911 = Cr$, the chord of half the arc of the *circular* segment *reCr*. Secondly, $\sqrt{re^2 + PC^2} = \sqrt{24^2 + 4^2} = \sqrt{592} = 24{\cdot}33105$. Then $(\overline{24{\cdot}33105 \times 2 + Cr}) \times PC \times \frac{4}{15} = (\overline{24{\cdot}33105 \times 2 + 12{\cdot}64911}) \times 4 \times \frac{4}{15} = 65{\cdot}398624$, the area of the *circular* segment *reCr*.

Lastly, CD : AB :: *reCr* : NVCN, *viz.* 40 : 120 :: 65·398624 : 196·195872, the area of the elliptical segment = *Answer.*

By Rule 2. (*See* Rule, Prob. XVII. Exam. 1.) $PC \div CD = 4 \div 40 = {\cdot}100$, the tabular height, against which stands the *Area Seg.* ·040875; hence $\cdot040875 \times AB \times CD = {\cdot}040875 \times 120 \times 40 = 196{\cdot}2$, the area of the elliptical segment NVCN = *Answer.*

Ex. 3. *(See Fig* 8 *of this Key.)* By Rule 1 *$(AB - mB) \times mB = (35 - 10) \times 10 = 250 = am \times mc$ or $= am^2$, the square of half the chord of the *circular* segment *acBa*, whose height is $10 = mB$, and $2\sqrt{am^2} = 2\sqrt{250} = 15{\cdot}811388 \times 2 = 31{\cdot}622776 = ac$ the chord of the *circular* segment *acBa*. Then, by Rule 2, Prob. XVI. First, $\sqrt{am^2 + mB^2} = \sqrt{250 + 100} = \sqrt{350} = 18{\cdot}7082869 = aB$, the chord of half the arc of the *circular* segment *acBa*. Secondly, $\sqrt{ac^2 + mB^2} = \sqrt{31{\cdot}622776^2 + 10^2} = \sqrt{1100} = 33{\cdot}1662479$. Then $(\overline{33{\cdot}1662479 \times 2 + aB}) \times mB \times \frac{4}{15} = (\overline{33{\cdot}1662479 \times 2 + 18{\cdot}7082869}) \times 10 \times \frac{4}{15} = 226{\cdot}77542$, the area of the *circular* segment *acBa*.

Lastly, AB : CD :: *acBa* : GHBG, *viz.* 35 : 25 :: 226·77542 : 161·982442, the area of the elliptical segment = *Answer.*

By Rule 2. (*See* Rule, and Note Prob. XVII. Exam. 3.) $mB \div AB = 10 \div 35 = {\cdot}285\frac{5}{7}$, the tabular height, the

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* Euclid, 35 p. iii. b.

Area Seg. corresponding to which is ·185166; then $\cdot185166 \times AB \times CD = \cdot185166 \times 35 \times 25 = 162{\cdot}02025$, the area of the elliptical segment GHBG = *Answer.*

Ex. 4. (*See Fig* 8 *of this Key.*) By Rule 1, *$(CD - PC) \times PC = (25-5) \times 5 = 100 = rP \times Pe$, or $= rP^2$, the square of half the chord of a *circular* segment whose height is $5 = PC$, and $2\sqrt{rP^2} = 2\sqrt{100} = 10 \times 2 = 20 = re$, the chord of the *circular* segment *reCr*. Then, by Rule 2, Prob. XVI. First, $\sqrt{rP^2 + PC^2} = \sqrt{100 + 25} = \sqrt{125} = 11{\cdot}180339 = Cr$, the chord of half the arc of the *circular* segment *reCr*. Secondly, $\sqrt{re^2 + PC^2} = \sqrt{20^2 + 5^2} = \sqrt{425} = 20{\cdot}615528$, Then $(20{\cdot}615528 \times 2 + Cr) \times PC \times \frac{4}{15} = (20{\cdot}615528 \times 2 + 11{\cdot}180339) \times 5 \times \frac{4}{15} = 69{\cdot}88186$, the area of the *circular* segment *reCr*.

Lastly, CD : AB :: *reCr* : NVCN, *viz.* 25 : 35 :: 69·88186 : 97·8346, the area of the elliptical segment = *Answer.*

By Rule 2. (*See* Rule, Prob. XVII. Exam. 1.) $PC \div CD = 5 \div 25 = \cdot200$, the tabular height, against which stands the *Area Seg.* ·111823; hence $\cdot111823 \times AB \times CD = \cdot111823 \times 35 \times 25 = 97{\cdot}845125$, the area of the elliptical segment NVCN = *Answer.*

Ex. 5. (*See Fig.* 8 *of this Key.*) By Rule 1, *$(AB - mB) \times mB = (70-20) \times 20 = 1000 = am \times mc$, or $= am^2$, the square of half the chord of the *circular* segment *acBa*, whose height is $20 = mB$, and $2\sqrt{am^2} = 2\sqrt{1000} = 31{\cdot}622776 \times 2 = 63{\cdot}245552 = ac$, the chord of the *circular* segment *acBa*. Then, by Rule 2, Prob. XVI. First, $\sqrt{am^2 + mB^2} = \sqrt{1000 + 400} = \sqrt{1400} = 37{\cdot}416573 = aB$, the chord of half the arc of the *circular* segment

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* Euclid, 35 p. iii. b.

acBa. Secondly, $\sqrt{ac^2 + mB^2} = \sqrt{63{\cdot}245552^2 + 20^2} = \sqrt{4400} = 66{\cdot}332495$. Then $(\overline{66{\cdot}332495 \times 2 +} aB) \times mB \times \frac{4}{15} = (\overline{66{\cdot}332495 \times 2} + 37{\cdot}416573) \times 20 \times \frac{4}{15} = 907{\cdot}101669$, the area of the *circular* segment *acBa*.

Lastly, AB : CD :: *ac*B*a* : GHBG, *viz* 70 : 50 :: 907·101669 : 647·929763, the area of the elliptical segment = *Answer*.

By Rule 2. (*See* Rule and Note Prob. XVII. Exam. 3.) $mB \div AB = 20 \div 70 = {\cdot}285\frac{5}{7}$ the tabular height, the *Area Seg.* corresponding to which is ·185166; then $\cdot185166 \times AB \times CD = \cdot185166 \times 70 \times 50 = 648{\cdot}081$, the area of the elliptical segment GHBG = *Answer*.

PROBLEM XXIV.*

Given the two Diameters of an Ellipse to find the Circumference.

Ex. 2. By Rule 1. $(AB + CD) \times 1{\cdot}5708 = (50 + 40) \times 1{\cdot}5708 = 141{\cdot}372$ yards, the circumference of the ellipse ACBD = *Answer*.

By Rule 2, $\frac{1}{2}(AB + CD) = \frac{1}{2}(50 + 40) = 45$ yards, the half sum of the two diameters. Then $\sqrt{\frac{1}{2}(AB^2 + CD^2)} = \sqrt{\frac{1}{2}(50^2 + 40^2)} = \sqrt{2050} = 45{\cdot}27692569$ yards. Then $(45{\cdot}27692569 + 45) \times 1{\cdot}5708 = 141{\cdot}806994873852$ yards, the circumference of the ellipse ACBD = *Answer*.

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* See the Diagram to Prob. XXIII.

By Rule 3. $1 - (40^2 \div 50^2) = 1 - {\cdot}64 = {\cdot}36 = d.$

$$\text{The second term } \frac{d}{2{\cdot}2} = \frac{{\cdot}36}{2\times 2} = \;.\;\;.\;\;.\;\;.\;\;.\;\;.\; A = {\cdot}090000$$

$$\text{The third term } \frac{3\,d}{4{\cdot}4} \times A = \frac{3 \times {\cdot}36}{4 \times 4} \times {\cdot}090000 = B = {\cdot}006075$$

$$\text{The fourth term } \frac{3{\cdot}5\,d}{6{\cdot}6} \times B = \frac{3 \times 5 \times {\cdot}36}{6 \times 6} \times {\cdot}006075 = C = {\cdot}000911$$

$$\text{The fifth term } \frac{5{\cdot}7\,d}{8{\cdot}8} \times C = \frac{5 \times 7 \times {\cdot}36}{8 \times 8} \times {\cdot}000911 = D = {\cdot}000179$$

$$\text{The sixth term } \frac{7{\cdot}9\,d}{10{\cdot}10} \times D = \frac{7 \times 9 \times {\cdot}36}{10 \times 10} \times {\cdot}000179 = E = {\cdot}000040$$

$$\text{The seventh term } \frac{9{\cdot}11\,d}{12{\cdot}12} \times E = \frac{9 \times 11 \times {\cdot}36}{12 \times 12} \times {\cdot}000040 = F = {\cdot}000009$$

$$\text{The eighth term } \frac{11{\cdot}13\,d}{14{\cdot}14} \times F = \frac{11 \times 13 \times {\cdot}36}{14 \times 14} \times {\cdot}000009 = G = {\cdot}000002$$

$$\text{The sum} = {\cdot}097216$$

Then $1 - {\cdot}097216 = {\cdot}902784$ the value of the series; hence $\cdot902784 \times 50 \times 3{\cdot}1416 = 141{\cdot}80931072$, the circumference of the ellipse = *Answer as before nearly.*

Ex. 3. By Rule 1. $\overline{AB+CD}\times 1{\cdot}5708 = \overline{70+50}\times 1{\cdot}5708 = 188{\cdot}496$, the circumference of the ellipse ACBD = *Answer.*

By Rule 2. $\frac{1}{2}(AB + CD) = \frac{1}{2}(70 + 50) = 60$, the half sum of the two diameters. Then $\sqrt{\frac{1}{2}(AB^2+CD^2)} = \sqrt{\frac{1}{2}(70^2 + 50^2)} = \sqrt{3700} = 60{\cdot}82762$. Then $\overline{60{\cdot}82762 + 60}\times 1{\cdot}5708 = 189{\cdot}796025496$, the circumference of the ellipse ACBD = *Answer.*

PROBLEM XXV.*

Any three of the four following Quantities being given to find the fourth: viz. an Ordinate, and its Abscissas, and the two Axes.

CASE I.

When the transverse, conjugate, and abscissas are given, to find the ordinate.

Ex. 2. AB : CD :: $\sqrt{(AB - mB)\times mB}$: G*m*, *viz.* 80 : 60 :: $\sqrt{(80-16)\times 16} : \frac{60\sqrt{(80-16)\times 16}}{80} = \frac{60\sqrt{64\times 16}}{80} = \frac{60\times 8\times 4}{80} = 24$, the length of the ordinate = *Answer.*

CASE 2.

When the transverse axis, conjugate, and ordinate are given, to find the abscissas.

Ex. 2. $\sqrt{CD^2 - 4\,Gm^2} = \sqrt{60^2 - (24^2\times 4)} = \sqrt{1296} = 36$, which ÷ by 2 = 18.
Then, as CD : AB :: 18 : O*m*, *viz.* 60 : 80 :: 18 :

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* See the Diagram to Prob. XXIII.

$\frac{80 \times 18}{60} = 24$, the distance of the ordinate from the centre, O, of the ellipse ACBD.

$$\left.\begin{array}{l} \therefore \tfrac{1}{2}\,AB + Om = (80 \div 2) + 24 = 64 \\ \quad \text{the greater abscissa } Am. \\ \text{And } \tfrac{1}{2}\,AB - Om = (80 \div 2) - 24 = 16 \\ \quad \text{the less abscissa } Bm. \end{array}\right\} = \textit{Answer}.$$

CASE III.

When the conjugate axis, ordinate, and abscissas are given, to find the transverse.

Ex. 2. $\frac{1}{2}\,CD = (60 \div 2) = 30$, the semi-conjugate axis OC. Then $\sqrt{OC^2 - Gm^2} = \sqrt{30^2 - 24^2} = \sqrt{324} = 18$, the square root of the difference of the squares of the semi-conjugate axis and the ordinate; hence $OC - 18 = 30 - 18 = 12$ the difference. Then, as $Gm^2 : CD \times Am :: 12 : AB$, *viz.* $24^2 : 60 \times 64 :: 12 : 80$, the length of the transverse axis = *Answer.*

CASE IV.

When the transverse axis, ordinate, and abscissas are given, to find the conjugate axis.

Ex. 2. $AB - Am = 80 - 64 = 16$, the less abscissa, mB.

Then $\sqrt{Am \times mB} = \sqrt{64 \times 16} = \sqrt{1024} = 32$, the square root of the product of the two abscissas.

Then as $32 : Gm :: AB : CD$, *viz.* $32 : 24 :: 80 : 60$, the length of the conjugate axis = *Answer.*

2. OF THE PARABOLA.

PROBLEM XXVI.

To find the Area of a Parabola, its Base and Height being given.

Ex. 2. $(\mathrm{HI} \times m\mathrm{O}) \times \frac{2}{3} = (16 \times 10) \times \frac{2}{3} = 106\frac{2}{3}$, the area of the parabola HIOH = *Answer.*

Ex. 3. $(\mathrm{CE} \times \mathrm{GO}) \times \frac{2}{3} = (38 \times 12) \times \frac{2}{3} = 304$, the area of the parabola CEOC = *Answer.*

PROBLEM XXVII.

To find the Area of the Zone of a Parabola, or of the Space included between two parallel double Ordinates.

Ex. 2. $\frac{(\mathrm{CE}^2 + \mathrm{HI}^2) + (\mathrm{CE} \times \mathrm{HI})}{\mathrm{CE} + \mathrm{HI}} = \frac{(25{\cdot}298^2 + 16^2) + (25{\cdot}298 \times 16)}{25{\cdot}298 + 16} = \frac{1300{\cdot}756804}{41{\cdot}298}$ $= 31{\cdot}4968$ the *quotient,* and $31{\cdot}4968 \times (\mathrm{G}m \times \frac{2}{3}) = 31{\cdot}4968 \times (9 \times \frac{2}{3}) = 188{\cdot}9808$, the area of the zone CEIH = *Answer.*

Ex. 3. $\frac{(\mathrm{AB}^2 + \mathrm{CE}^2) + (\mathrm{AB} \times \mathrm{CE})}{\mathrm{AB} + \mathrm{CE}} = \frac{(24^2 + 20^2) + (24 \times 20)}{24 + 20} = \frac{1456}{44} = 33\frac{1}{11}$ the *quotient,* and $33\frac{1}{11} \times (\mathrm{DG} \times \frac{2}{3}) = 33\frac{1}{11} \times (5{\cdot}5 \times \frac{2}{3}) = 121\frac{1}{3}$, the area of the zone ABEC = *Answer.*

PROBLEM XXVIII.

To find the Length of any Arc of a Parabola, cut off by a double Ordinate.

Ex. 2. $\sqrt{\mathrm{H}m^2 + \frac{4}{3}\, m\mathrm{O}^2} = \sqrt{8^2 + (\frac{4}{3} \times 3^2)} = \sqrt{76} = 8{\cdot}71779$ = HO, half the length of the curve *nearly.*

Then $(\mathrm{H}m^2 + \frac{2}{3}\,m\mathrm{O}^2) \times \frac{4}{\mathrm{H}m} = 8^2 + (\frac{2}{3} \times 3^2) \times \frac{4}{8}$ $= \frac{280}{8} = 35$, the *quotient*. Then $\overline{(\mathrm{HO} \times 9} - 35) \times \frac{2}{5} = (\overline{8{\cdot}71779 \times 9} - 35) \times \frac{2}{5} = 17{\cdot}38404$, the length of the curve HOI = *Answer*.

PROBLEM XXIX.

In a Parabola, any three of the four following Terms being given, viz. any two Ordinates and their two Abscissas, to find a fourth.

Ex. 2. $\mathrm{H}m^2 : \mathrm{O}m :: \mathrm{CG}^2 : \mathrm{GO}$, *viz.* $6^2 : 9 :: 8^2, : 16$, the abscissa = *Answer.*

Ex. 3. *First*, for the ordinate corresponding to the abscissa DO = 9. Then as $m\mathrm{O} : \mathrm{H}m^2 :: \mathrm{DO} : \mathrm{AD}^2$, *viz.* $4 : 8^2 :: 9 : 144$, the square of the ordinate; $\therefore$ $\sqrt{\mathrm{AD}^2} = \sqrt{144} = 12$, the ordinate AD.

Second, for the ordinate corresponding to the abscissa $m\mathrm{O} = 4$. Then as $\mathrm{DO} : \mathrm{AD}^2 :: m\mathrm{O} : \mathrm{H}m^2$, *viz.* $9 : 12^2 :: 4 : 64$, the square of the ordinate; $\therefore$ $\sqrt{\mathrm{H}m^2} = \sqrt{64} = 8$, the ordinate H*m*.

Third, for the abscissa corresponding to the ordinate AD = 12. Then as $\mathrm{H}m^2 : m\mathrm{O} :: \mathrm{AD}^2 : \mathrm{DO}$, *viz.* $8^2 : 4 :: 12^2 : 9$, the abscissa.

Fourth, for the abscissa corresponding to the ordinate H*m* = 8. Then as $\mathrm{AD}^2 : \mathrm{DO} :: \mathrm{H}m^2 : m\mathrm{O}$, *viz.* $12^2 : 9 :: 8^2 : 4$, the abscissa.

PROBLEM XXX.

Given two Ordinates perpendicular to the Axis, and their Distance to find the corresponding Abscissas.

Ex. 2. $\mathrm{AD}^2 - \mathrm{CG}^2 : \mathrm{DG} :: \mathrm{AD}^2 : \mathrm{DO}$, *viz.* $10^2 - 6^2 : 6 :: 10^2 : 9{\cdot}375$, the greater abscissa = 1*st Answer.*

$AD^2 - CG^2 : DG :: CG^2 : GO$, *viz.* $10^2 - 6^2 : 6 :: 6^2 : 3{\cdot}375$, the less abscissa $=$ *2nd Answer.*

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3. OF THE HYPERBOLA.

PROBLEM XXXI.

Given the transverse Axis of an Hyperbola, the Conjugate, and the Abscissa, to find the Area.

Ex. 2. *First,* $\sqrt{(AB \times EA) + \frac{5}{7}EA^2} \times 21 = \sqrt{(30 \times 10) + (\frac{5}{7} \times 10^2)} \times 21 = 19{\cdot}2724 \times 21 = 404{\cdot}7204$ the *product.*

Second, $4\sqrt{AB \times EA} = 4\sqrt{30 \times 10} = 17{\cdot}3205 \times 4 = 69{\cdot}282$. Then $(69{\cdot}282 + 404{\cdot}7204) \div 75 = 6{\cdot}32$, the *quotient.*

Third, $(CD \times EA \times 4) \div AB = (18 \times 10 \times 4) \div 30 = 24$, the *quotient.* Then $24 \times 6{\cdot}32 = 151{\cdot}68$ the area of the hyperbola PAG $=$ *Answer.*

Ex. 3. *First,* $\sqrt{(AB \times BW) + \frac{5}{7}BW^2} \times 21 = \sqrt{(50 \times 25) + (\frac{5}{7} \times 25^2)} \times 21 = 41{\cdot}1877 \times 21 = 864{\cdot}9417$ the *product.*

Second, $4\sqrt{AB \times BW} = 4\sqrt{50 \times 25} = 35{\cdot}3553 \times 4 = 141{\cdot}4212$. Then $(141{\cdot}4212 + 864{\cdot}9417) \div 75 = 13{\cdot}4181$ the *quotient.*

Third, $(mBn \times BW \times 4) \div AB = (30 \times 25 \times 4) \div 50 = 60$, the *quotient.*

Then $60 \times 13{\cdot}4181 = 805{\cdot}086$, the area of the hyperbola TBS $=$ *Answer.*

PROBLEM XXXII.

To find the Length of any Arc of an Hyperbola, beginning at the Vertex.

Ex. 2. *First,* $CD^2 \div AB = 36^2 \div 60 = 21{\cdot}6$, the latus rectum or parameter LR.

Second, (19 AB + 21 LR) × EA = (19 × 60) + (21 × 21·6) × 20 = 31872, *the product*.

Again, (9 AB + 21 LR) × EA = (9 × 60) + (21 × 21·6) × 20 = 19872, *the product*.

Then 31872 + 15 CD^2 = 31872 + (15 × 36^2) = 51312, the greater sum.

And 19872 + 15 CD^2 = 19872 + (15 × 36^2) = 39312, the less sum.

Third, as 39312 : 51312 :: 24 : 31·326 = PLA, half the length of the curve, which × by 2 = 62·652, the length of the whole curve PLARG = *Answer*.

Ex. 3. *First*, CD^2 ÷ AB = 20^2 ÷ 30 = 13·3′, the latus rectum or parameter LR.

Second, (19 AB + 21 LR) × EA = (19 × 30) + (21 × 13·3′) × 6 = 5100, *the product*.

Again, (9 AB + 21 LR) × EA = (9 × 30) + (21 × 13·3′) × 6 = 3300, *the product*.

Then 5100 + 15 CD^2 = 5100 + (15 × 20^2) = 11100 the greater sum.

And 3300 + 15 CD^2 = 3300 + (15 × 20^2) = 9300, the less sum.

Third, as 9300 : 11100 :: 9·797959 : 11·694338 = PLA, half the length of the curve, which × by 2 = 23·388676, the length of the whole curve PLARG = *Answer*.

PROBLEM XXXIII.

In an Hyperbola, any three of the four following Terms being given, viz. the transverse and conjugate Diameters, an Ordinate and its Abscissa, to find the fourth.

CASE I.

To find the Ordinate.

Ex. 2. EA + AB = 8 + 24 = 32, the greater abscissa EB.

Then as AB : CD :: $\sqrt{EA \times EB}$: PE, *viz.* 24 : 21 :: $(\sqrt{8 \times 32}) = 16$: 14, the length of the ordinate = *Answer.*

Ex. 3. WB + AB = 40 + 120 = 160, the greater abscissa WA.

Then as AB : mBn :: $\sqrt{WB \times WA}$: TW, *viz.* 120 : 72 :: $(\sqrt{40 \times 160}) = 80$: 48, the length of the ordinate = *Answer.*

CASE II.

To find the two Abscissas.

Ex. 2. $\sqrt{mBn^2 + 4\,TW^2} = \sqrt{72^2 + (4 \times 48^2)} = \sqrt{14400} = 120$, the half = 60.

Then as mBn : AB :: 60 : WO, *viz.* 72 : 120 :: 60 : 100, the distance of the ordinate from the centre of the transverse axis.

Now WO − $\frac{1}{2}$ AB = 100 − 60 = 40, the less abscissa WB = 1*st Answer.*

And WO + $\frac{1}{2}$ AB = 100 + 60 = 160, the greater abscissa WA = 2*nd Answer.*

Ex. 3. $\sqrt{CD^2 + 4\,PE^2} = \sqrt{18^2 + (4 \times 12^2)} = \sqrt{900} = 30$, the half = 15.

Then as CD : AB :: 15 : EO, *viz.* 18 : 30 :: 15 : 25, the distance of the ordinate from the centre of the transverse axis.

Now EO − $\frac{1}{2}$ AB = 25 − 15 = 10, the less abscissa EA = 1*st Answer.*

And EO + $\frac{1}{2}$ AB = 25 + 15 = 40, the greater abscissa EB = 2*nd Answer.*

CASE III.

To find the conjugate Axis.

Ex. 2. EA + AB = 20 + 60 = 80, the greater abscissa EB.

Then as $\sqrt{EA \times EB}$: PE :: AB : CD, *viz.* $(\sqrt{20 \times 80}) = 40$: 24 :: 60 : 36, the conjugate axis = *Answer.*

Ex. 3. WB + BA = 40 + 120 = 160, the greater abscissa WA.

Then as $\sqrt{WA \times WB}$: TW :: AB : *m*B*n*, *viz.* $(\sqrt{160 \times 40}) = 80$: 48 :: 120 : 72, the conjugate axis = *Answer.*

CASE IV.

To find the transverse Axis.

Ex. 2. $\sqrt{(CD^2 + 4\,PE^2)} - CD = \sqrt{(18^2 + 4 \times 12^2)} - 18 = 30 - 18 = 12$, the half = 6.

Then as PE^2 : 6 :: CD × EB : AB, *viz.* 12^2 : 6 :: 18 × 40 : 30, the length of the transverse axis = *Answer.*

Ex. 3. $\sqrt{(mBn^2 + 4TW^2)} + mBn = \sqrt{(72^2 + 4 \times 48^2)} + 72 = 120 + 72 = 192$, the half = 96.

Then as TW^2 : 96 :: *m*B*n* × WB : AB, *viz.* 48^2 : 96 :: 72 × 40 : 120, the transverse axis = *Answer.*

PROBLEM XXXIV.

To find the Area of a Curve by equidistant Ordinates, the Base being divided into any even Number of equal Parts.

Ex. 2. Here 0 + 1 = 1, the sum of the first and last ordinates. Then (·4358899 + ·7141428 + ·8660254 + ·9539392 + ·9949874) × 4 = 15·8599388, four times the sum of the even ordinates. Then (·6 + ·8 + ·9165151 + ·9797959) × 2 = 6·592622, twice the sum of the odd ordinates. Hence $\frac{1}{3}$ (1· + 15·8599388

$+ 6·592622) \times \frac{1}{10} = ·78175202$, the area of the quadrant = *Answer*.

Ex. 3. $14 + 25 = 39$ feet, the sum of the first and last ordinates.

Then $(15+17+20+23) \times 4 = 300$ feet, four times the sum of the even ordinates.

Then $(16+18+22) \times 2 = 112$ feet, twice the sum of the odd ordinates.

Hence $\frac{1}{3}(39+300+112) \times 2 = 300\frac{2}{3}$ square feet, the area of the irregular figure = *Answer*.

Ex. 4. $AD + GH = 5 + 8 = 13$, the sum of the first and last ordinates.

Then $4\,EF = 4 \times 7 = 28$, four times the sum of the even ordinate; hence $\frac{1}{3}(13 + 28) \times 5 = 68\frac{1}{3}$, the area of the figure AGHD = *Answer*.

Ex. 5. $\frac{10}{10} + \frac{10}{20} = 1 + ·5 = 1·5$, the sum of the first and last ordinates.

Then $(\frac{10}{11} + \frac{10}{13} + \frac{10}{15} + \frac{10}{17} + \frac{10}{19}) \times 4 = (·9'0' + ·7'69230' + ·6' + ·588235294 + ·526315789) \times 4 = 13·838157708$, four times the sum of the even ordinates.

Then $(\frac{10}{12} + \frac{10}{14} + \frac{10}{16} + \frac{10}{18}) \times 2 = (·83' + ·7'14285' + ·625 + ·5') \times 2 = 5·456349204$, twice the sum of the odd ordinates; hence $\frac{1}{3}(1·5 + 13·838157708 + 5·456349204) \times \frac{1}{10} = ·6931502304$, the area of the curve = *Answer*.

Ex. 6. Here the middle ordinate, being the abscissa, is 2, the first and last ordinates are each = 0, and the common distance = 6; hence the area is thus found $\frac{1}{3}(4 \times 2) \times 6 = 16$, the area = *Answer*.

MENSURATION OF SOLIDS.

PROBLEM I.

To find the solidity of a Cube.

Ex. 2. $AB^3 = 17{\cdot}5^3 = 5359{\cdot}375$ solid or cubic inches, which ÷ by $1728 = 3{\cdot}10149$ solid or cubic feet ABCDEFG = *Answer.*

Ex. 3. $AB^3 = \overline{19 \text{ ft. } 4 \text{ in.}}|^3 = 7226$ ft. 4 in. 5 sec. 4 thirds the solidity of the cube ABCDEFG = *Answer.*

Ex. 4. $AB^3 = \overline{10\frac{1}{3}}|^3 = 1103\frac{10}{27}$ solid feet the content of the cellar ABCDEFG.

Then as 1 solid yd. : $1103\frac{10}{27}$ solid ft. :: 1*s.* : £2. 0*s.* $10\frac{94}{243}$*d.*, the expense of digging the cellar = *Answer.*

Ex. 5. $\frac{12^3}{3^3} = \frac{1728}{27} = 64$, the number of three-inch cubes = *Answer.*

PROBLEM II.

To find the Solidity of a Parallelopipedon.

Ex. 2. AB × AG × AD = 25 ft. × 2 ft. 1 in. × 9 in. = 39 ft. 0 in. 9 sec. the solid content of the timber ABCDEFG = *Answer.*

Ex. 3. AB × AG × AD = 18 ft. × 1·25 ft. × 1·25 ft. = 28·125 ft. the solidity of the timber ABCDEFG.

Then, as 28·125 solid ft. : 1 solid ft. :: 18 lineal ft.

: 7·68 lineal inches to be cut off, which will make a solid foot = *Answer.*

Ex. 4. AB × AG × AD = 16·75 ft. × 1·583 ft. × 2·75 ft. = 72·93075625 solid feet, the content of the timber ABCDEFG.

Then, as 72·93 solid ft. : 1 solid ft. :: 16·75 lineal ft. : 2·756 inches in length, which make a solid foot = *Answer.*

PROBLEM III.

To find the Solidity of a Prism.

Ex. 2. Here 12 ft. 5 in. = 149 inches. Then, by Rule 2, and Table, Prob. X. Superfices. $17^2 \times {\cdot}433013 = 125{\cdot}140757$ sq. inches, the area of the base. Then, by the Rule, $\frac{125{\cdot}140757 \times 149 \text{ in.}}{1728} = 10{\cdot}790493$ solid feet = 1*st Answer.*

To find the Superficies.

By the Note. 149 in. × 17 in. × 3 = 7599 square inches, the area of the three sides.

Hence $\frac{(125{\cdot}140757 \times 2) + 7599}{144} = 54{\cdot}508899$ sq. feet, the superficies of the whole figure = 2*nd Answer.*

Ex. 3. By Note to Rule 1, Prob. X. Superficies. 28 ÷ by 2 = 14 inches each side of the base of the prism: Then, by Rule 2, and Table, Prob. X. Superficies. $14^2 \times 2{\cdot}598076 = 509{\cdot}222896$ sq. inches, the area of the base. Then, by the Rule, $\frac{509{\cdot}222896 \times 134 \text{ in.}}{1728}$ = 39·488349 cubic feet, the solidity of the hexagonal prism = *Answer.*

To find the Superficies.

By the Note. 134 in. × 14 in. × 6 = 11256 sq. in., the upright surface; hence $\frac{(509{\cdot}222896 \times 2) + 11256}{144}$ $=$ 85·239206 sq. feet, the surface of the whole figure $=$ *Answer.*

Ex. 4. 50 in. $\div$ 10 $=$ 5 in., one of its sides. Then, by Rule 2, and Table, Prob. X. Superficies, $5^2 \times 7{\cdot}694209 = 192{\cdot}355225$ sq. inches, the area of the base. Then, by the Rule, $\frac{192{\cdot}355225 \times 30 \text{ ft.}}{144} =$ 40·074 cubic feet, the solidity of the decagonal prism $=$ *Answer.*

To find the Superficies.

By the Note. 50 in. $\times$ 360 in. $=$ 18000 square inches, the area of the upright surface; hence $\frac{(192{\cdot}355225 \times 2) + 18000}{144} = 127{\cdot}6716$ square feet, the surface of the whole figure $=$ *Answer.*

Ex. 5. Here 48 in. $=$ 4 feet. Hence 4 ft. $\times$ 12 ft. $\times$ 10 $=$ 480 sq. feet, the area of the ten octagonal prisms. Then, as 1 sq. yd. : 480 sq. ft. :: 10*d.* : £2. 4*s.* 5⅓*d.*, the expense of painting the whole $=$ *Ans.*

Ex. 6. By Rule, Prob. VIII. Superficies. ½ (20 yds. + 16 yds.) $\times$ 10 yds. $=$ 180 sq. yards, the area of the end of the canal. Then, by the Rule, 180 $\times$ 50 yds. $=$ 9000 solid yards of earth $=$ *Answer.*

PROBLEM IV.

To find the Solidity of a Cylinder.

Ex. 2. By Rule 5, Prob. XIII. Superficies. $8^2 \times {\cdot}07958 = 5{\cdot}09312$ square feet, the area of the base. Then, by the Rule, $5{\cdot}09312 \times 18 \text{ ft.} = 91{\cdot}67616$ solid feet, the content of the timber $=$ *Answer.*

Ex. 3. By Rule 5, Prob. XIII. Superficies. $86^2 \times {\cdot}07958 = 588{\cdot}57368$ sq. inches, the area of the base.

Then, by the Rule, $\frac{588{\cdot}57368 \times 240 \text{ in.}}{1728} = 81{\cdot}74634$ solid feet, the content of the timber = *Answer.*

Ex. 4. By Rule 2, Prob. XIII. Superficies. $20{\cdot}75^2 \times {\cdot}7854 = 338{\cdot}1637875$ sq. inches, the area of the base. Then, by the Rule, $\frac{338{\cdot}1637875 \times 55 \text{ in.}}{1728} =$ 10·7633149 solid feet, the content of the cylinder = *Answer.*

To find the Superficies.

By Rule 1. Prob. XI. Superficies. $20{\cdot}75 \text{ in.} \times 3{\cdot}1416 = 65{\cdot}1882$ inches, the circumference of the base; and, by the Note, $65{\cdot}1882 \times 55 \text{ in.} = 3585{\cdot}351$ square inches, the upright or convex surface. Then, by Rule 2, Prob. XIII. Superficies. $20{\cdot}75^2 \times {\cdot}7854 \times 2 = 676{\cdot}327575$ sq. inches, the area of both ends. Lastly, $\frac{3585{\cdot}351 + 676{\cdot}327575}{144} = 29{\cdot}59499$ sq. feet, the surface of the whole figure = *Answer.*

Ex. 5. Here 3 cubic feet = 5184 cubic inches. Then, by Rule 5, Prob. XIII. Superficies. $44^2 \times {\cdot}07958 = 154{\cdot}06688$ sq. inches, the area of one of its ends; hence $\frac{5184}{154{\cdot}06688} = 33{\cdot}64$ inches, the length of the piece to be cut off = *Answer..*

Ex. 6. By Rule, Prob. XIX. Superficies. $20 \text{ in.} - (1\frac{1}{2} \text{ in.} \times 2) = 17$ inches, the inside diameter; hence $(20 + 17) \times (20 - 17) \times {\cdot}7854 = 87{\cdot}1794$ square inches, the area of the metal at one of its ends; and $87{\cdot}1794 \times 50 = 4358{\cdot}97$ cubic inches, the solidity of the metal = 1*st Answer.*

Then, as 1 cub. in. : 4358·97 cub. in. :: $4\frac{1}{4}$ oz. : 1157·8514 pounds, the weight of the iron roller = 2*nd Answer.*

Lastly, as 1 lb. : 1157·8514 lbs. :: $3\frac{1}{4}$*d.* : £15. 13*s.* 7*d.* ·017, the value of the roller = 3*rd Answer.*

PROBLEM V.

To find the Solidity of a Pyramid.

Ex. 2. By Rule 2, and Table, Prob. X. Superficies. $21{\cdot}5^2 \times {\cdot}433013 = 200{\cdot}16025925$ sq. inches, the area of the base. Then, by the Rule, $\frac{200{\cdot}16025925 \times (16\text{ft} \div 3)}{144} = 7{\cdot}41334293$ solid feet, the content of the triangular pyramid = *Answer.*

To find the Superficies.

By the Note. (*See also* Rule 2, and Table, Prob. X. Superficies.) $21{\cdot}5$ in. $\times\ {\cdot}2886751 = 6{\cdot}20651465$ in. = OD, the perpendicular from the centre of the triangular base to the middle of one of its sides. Then, *$\sqrt{6{\cdot}20651465^2 + 192^2} = \sqrt{36902{\cdot}5208241006646225}$ $= 192{\cdot}10028845$ inches, the slant height. Then $\frac{1}{2}(21{\cdot}5\text{in.} \times 3) = 32{\cdot}25$ inches, half the circumference of the base, which × by $192{\cdot}10028845 = 6195{\cdot}2343025125$ square inches, the surface of the sides; hence $\frac{6195{\cdot}2343025125 + 200{\cdot}16025925}{144} = 44{\cdot}4124622344$ square feet, the whole superficies = *Answer.*

Ex. 3. By Rule 2, and Table, Prob. X. Superficies. $15^2 \times 3{\cdot}633912 = 817{\cdot}6302$ square inches, the area of the base. Then, by the Rule, $\frac{817{\cdot}6302 \times (13{\cdot}5\text{ft.} \div 3)}{144} = 25{\cdot}55094$ solid feet, the content of the heptagonal pyramid = *Answer.*

To find the Superficies.

By the Note. (*See also* Rule 2, and Table, Prob. X. Superficies.) 15 in. × $1{\cdot}0382617 = 15{\cdot}5739255$ in, = OD, the perpendicular from the centre of the heptagonal base to the middle of one of its sides.

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* Euclid, 47 p. i. b.

Then [*] $\sqrt{15{\cdot}5739255^2 + 162^2} = \sqrt{26486{\cdot}547155479555025} = 162{\cdot}7468806$ inches, the slant height.

Then, $\frac{1}{2}(15 \text{ in.} \times 7) = 52{\cdot}5$ inches, half the circumference of the base; and $162{\cdot}7468806 \times 52{\cdot}5 = 8544{\cdot}2112315$ square inches, the surface of the sides: hence $\frac{8544{\cdot}2112315 + 817{\cdot}6302}{144} = 65{\cdot}0127877$ sq. feet, the whole surface $=$ *Answer.*

Ex. 4. By Rule, Prob. V. Superficies. $\frac{1}{2}(13+14+15) = 21$ feet, the half sum;

$$\left.\begin{array}{l} 21 - 13 = 8 \\ 21 - 14 = 7 \\ 21 - 15 = 6 \end{array}\right\} \text{the three remainders.}$$

Then, $\sqrt{21 \times 8 \times 7 \times 6} = \sqrt{7056} = 84$ square feet, the area of the base.

Then, by the Rule, $84 \times (63 \text{ ft.} \div 3) = 1764$ cubic feet, the solidity of the triangular pyramid $=$ *Answer.*

Ex. 5. By Rule 2, and Table, Prob. X. Superficies. $10^2 \times 2{\cdot}598076 = 259{\cdot}8076$ square feet, the area of the base. Then, by the Rule, $259{\cdot}8076 \times (45 \text{ ft.} \div 3) = 3897{\cdot}114$ feet, the solidity of the hexagonal pyramid $=$ *Answer.*

To find the Superficies.

By Rule 2, and Table, Prob. X. Superficies. 10 ft. $\times \,{\cdot}8660254 = 8{\cdot}660254$ feet $=$ OD, the perpendicular from the centre of the hexagonal base to the middle of one of its sides.

Then [*] $\sqrt{8{\cdot}660254^2 + 45^2} = \sqrt{2099{\cdot}999999344516} = 45{\cdot}825756$ feet, the slant height.

Then $\frac{1}{2}(10 \text{ft.} \times 6) = 30$ feet, half the circumference of the base; and 30 ft. $\times\ 45{\cdot}825756 = 1374{\cdot}77268$ square feet, the surface of the sides; hence $1374{\cdot}77268$

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[*] **Euclid, 47 p. 1. b.**

$+ 259{\cdot}8076 = 1634{\cdot}58028$ square feet, the whole surface = *Answer.*

Ex. 6. $AB^2 = 3^2 = 9$ square feet, the area of the base ABCGA.

Then, by the Rule, $ABCGA \times \frac{1}{3} OV = 9 \times (24 \text{ ft.} \div 3) = 72$ solid feet, the content of the whole pyramid ABCGV = *Answer.*

To find the Superficies.

$\frac{1}{2} AB = 3 \div 2 = 1{\cdot}5$ feet $= OD$, the perpendicular from the centre of the base to the middle of its side. Then ${}^*\sqrt{OD^2 + OV^2} = \sqrt{1{\cdot}5^2 + 24^2} = \sqrt{578{\cdot}25} = 24{\cdot}0468$ feet, the slant height DV.

Then $AB \times 2 = 3 \times 2 = 6$ feet $= ABC$, half the circumference of the base ABCGA.

Now, by the Note, $\frac{1}{2} ABCGA \times DV = 6 \times 24{\cdot}0468 = 144{\cdot}2808$ square feet, the surface of the sides of ABCGV; which added to the surface of the base of $ABCGA = 144{\cdot}2808 + 9 = 153{\cdot}2808$ square feet, the superficies of the whole figure = *Answer.*

Ex. 7. $AB^2 = 2{\cdot}5^2 = 6{\cdot}25$ square feet, the area of the base ABCGA.

Then $\frac{1}{2} AB = 2{\cdot}5 \div 2 = 1{\cdot}25$ feet $= OD$, the perpendicular from the centre of the base to the middle of its side.

Then ${}^*\sqrt{DV^2 - OD^2} = \sqrt{21^2 - 1{\cdot}25^2} = \sqrt{439{\cdot}4375} = 20{\cdot}962764$ feet, the perpendicular height OV.

Then, by the Rule, $ABCGA \times \frac{1}{3} OV = 6{\cdot}25 \times (20{\cdot}962764 \text{ ft.} \div 3) = 43{\cdot}672425$ solid feet, the content of the pyramid ABCGV.

Then, as 1 solid ft. : 43·672425 solid ft. :: 7*s.* : £15. 5*s.* 8¼*d.* ·9348, the price of the pyramid.

Again.—To find the Superficies. By the Note. 2AB

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* Euclid, 47 p. i. b.

$= 2{\cdot}5 \times 2 = 5$ feet $=$ ABC, half the circumference of the base ABCGA. Now, $\frac{1}{2}$ ABCGA $\times$ DV $= 5 \times 21 = 105$ square feet, the upright surface of the pyramid ABCGV.

Then, as 1 sq. ft. : 105 sq. ft. :: 8*d.* : £3. 10*s.* the price for polishing the upright surface; hence £3. 10*s.* + £15. 5*s.* 8¼*d.* ·9348 = £18. 15*s.* 8¼*d.* ·9348, the expense of the stone when finished = *Answer.*

Ex. 8. 5 ft. 10 in. = 70 inches. Then, by Rule 2, and Table, Prob. X. Superficies. $70^2 \times 4{\cdot}828427 = 23659{\cdot}2923$ square inches, the area of the base.

Then, by the Rule, $\dfrac{23659{\cdot}2923 \times (45 \text{ feet} \div 3)}{144}$ $= 2464{\cdot}509614$ solid feet, the solidity of the whole octagonal pyramid = 1*st Answer.*

Again.—To find the solidity of the cavity. 4 ft. 11 in. = 59 inches. Then, by Rule 2, and Table, Prob X. Superficies. $59^2 \times 4{\cdot}828427 = 16807{\cdot}754387$ square inches, the area of the inner base.

Then, by the Rule, $\dfrac{16807{\cdot}754387 \times (41 \text{ feet} \div 3)}{144}$ $= 1595{\cdot}180393$ feet, the solidity of the cavity of the pyramid = 2*nd Answer.*

Hence $\dfrac{2464{\cdot}509614 - 1595{\cdot}180393}{27} = 32{\cdot}197378$ solid yards, the content of the stone-work = 3*rd Answer.*

PROBLEM VI.

To find the Solidity of a Cone.

Ex. 2. *(See Fig. 9 of this Key.)* By Rule 2, Prob. XIII. Superficies. $AB^2 \times {\cdot}7854 = 10^2 \times {\cdot}7854 = 78{\cdot}54$, the area of the base AGBH. Then, by the Rule, AGBH $\times \frac{1}{3}$ DC $= 78{\cdot}54 \times (68{\cdot}1 \div 3) = 1782{\cdot}858$, the solidity of the cone ADBCA = *Answer.*

To find the Superficies.

*$\sqrt{AD^2 + CD^2} = \sqrt{5^2 + 68{\cdot}1^2} = \sqrt{4662{\cdot}61} =$ 68·283306, the slant height AC.

Then, by Rule 1, Prob. XI. Superficies. AB × 3·1416 = 10 × 3·1416 = 31·416, the circumference of the base AGBH.

Then, by the Note. $\frac{1}{2}$ (AGBH × AC) = $\frac{1}{2}$ (31·416 × 68·283306) = 1072·594170648, the convex surface ACB.

Lastly, ACB + AGBH = 1072·594170648 + 78·54 = 1151·134170648, the superficies of the whole cone = *Answer.*

Ex. 3. *(See Fig.* 9 *of this Key.)* By Rule, Prob. XXI. Superficies. AB × GH × ·7854 = 15·2 × 10 × ·7854 = 119·3808, the area of the base AGBH.

Then, by the Rule. AGBH × $\frac{1}{3}$ DC = 119·3808 × (22 ÷ 3) = 875·4592, the solidity of the elliptical cone ADBCA = *Answer.*

Ex. 4. By Rule 5, Prob. XIII. Superficies. AGBH^2 × ·07958 = 9^2 × ·07958 = 6·44598 square feet, the area of the base AGBH.

Then, by the Rule. AGBH × $\frac{1}{3}$ DC = 6·44598 × (10·5 ft. ÷ 3) = 22·56093 solid feet, the content of the cone ADBCA = *Answer.*

To find the Superficies.

By Rule 2, Prob. XI. Superficies. AGBH × ·318309 = 9 ft. × ·318309 = 2·864781 feet, the diameter of the base ADB, which ÷ by 2 = 1·4323905 feet, half the diameter = AD.

Then, *$\sqrt{AD^2 + CD^2} = \sqrt{1{\cdot}4323905^2 + 10{\cdot}5^2}$ $= \sqrt{112{\cdot}30174254449025} =$ 10·5972516 feet, the slant height AC.

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* Euclid, 47 p. i. b.

Then, by the Note. $\frac{1}{2}$(AC × AGBH) = $\frac{1}{2}$(10·5972516 × 9) = 47·6876322 square feet, the convex surface ACB.

Lastly, ACB + AGBH = 47·6876322 + 6·44598 = 54·1336122 square feet, the whole surface of the cone = *Answer.*

Ex. 5. By Rule 5, Prob. XIII. Superficies. $\text{AGBH}^2 \times {\cdot}07958 = 24^2 \times {\cdot}07958 = 45{\cdot}83808$ square feet, the area of the base AGBH.

Now, by Rule 2, Prob. XI. Superfi. AGBH × ·318309 = 24 ft. × ·318309 = 7·639416 feet, the diameter of the base ADB, which ÷ by 2 = 3·819708 feet, half the diameter = AD. Then, $^*\sqrt{\text{AC}^2 - \text{AD}^2} = \sqrt{32^2 - 3{\cdot}819708^2} = \sqrt{1009{\cdot}409830794736} =$ 31·7712107 feet, the ⊥ height DC. Then, by the Rule. AGBH × $\frac{1}{3}$ DC = 45·83808 × (31·7712107 ft. ÷ 3) = 485·44376286528 solid feet, the content of the cone ADBCA = *Answer.*

To find the Superficies.

By the Note. $\frac{1}{2}$ (AGBH × AC) = $\frac{1}{2}$ (24 ft. × 32 ft.) = 384 square feet, the convex surface ACB. Then, ACB + AGBH = 384 + 45·83808 = 429·83808 sq. feet, the superficies of the whole cone = *Answer.*

Ex. 6. By Rule 2, Prob. XI. Superficies. AGBH × ·318309 = 64 ft. × ·318309 = 20·371776 feet, the diameter ADB, which ÷ by 2 = 10·185888 feet, half the diameter = AD. Then, $^*\sqrt{\text{AD}^2 + \text{CD}^2} = \sqrt{10{\cdot}185888^2 + 118^2} = \sqrt{14027{\cdot}752314348544} =$ 118·438812 feet, the slant height AC. Then, by the Note. $\frac{1}{2}$ (AGBH × AC) = $\frac{1}{2}$ (64 × 118·438812) = 3790·041984 square feet, the convex surface of the cone ACB = 1*st Answer.*

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* Euclid, 47 p. i. b.

Then, as 1 sq. yd. : 3790·041984 sq. ft. :: 8*d.* : £14. 0*s.* 8¾*d.* ·7048, the expense of painting the conical church spire = *2nd Answer.*

PROBLEM VII.

To find the Solidity of the Frustum of a Pyramid.

Ex. 2. By the General Rule. (*See* Rule 2, and Table, Prob. X. Superficies.) $25^2 \times {\cdot}433013 = 270{\cdot}633125$ square inches, the area of the greater end. *Again.*—by Rule 2, and Table, Prob. X. Superficies. $9^2 \times {\cdot}433013 = 35{\cdot}074053$ square inches, the area of the less end. Then, by the Rule, $\sqrt{270{\cdot}633125 \times 35{\cdot}074053} = \sqrt{9492{\cdot}200569805625} = 97{\cdot}427925$ square inches, the mean area, and $97{\cdot}427925 + 270{\cdot}633125 + 35{\cdot}074053 = 403{\cdot}135103$ square inches, the sum. Then, $\dfrac{403{\cdot}135103 \times (15\text{ ft.} \div 3)}{144} = 13{\cdot}997746$ solid feet, the content of the frustrum of the triangular pyramid = *Answer.*

To find the Superficies.

$25\text{ in.} \times 3 = 75$ inches, $\Big\}$ the perimeter of the greater end

and $9\text{ in.} \times 3 = 27$ inches, $\Big\}$ the perimeter of the less end;

The sum = 102 inches; which ÷ by 2 = 51 inches, the half sum.

Then, by the Note. (*See* Rule 2, and Table, Prob. X. Superficies.) $(25\text{ in.} - 9\text{ in.}) \times {\cdot}2886751 = 4{\cdot}6188016$ inches = PG.

Now, *$\sqrt{PG^2 + GO^2} = \sqrt{4{\cdot}6188016^2 + 180^2} = \sqrt{32421{\cdot}33332822016256} = 180{\cdot}0592494$ inches, the slant height PO.

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* Euclid, 47 p. i. b.

And, by the Note. $180{\cdot}0592494 \times 51$ ft. $=$ 9183·0217194 square inches, the surface of the sides.

Then, $\dfrac{9183{\cdot}0217194 + 270{\cdot}633125 + 35{\cdot}074053}{144} =$ 65·8939506 square feet, the surface of the whole frustum $=$ *Answer.*

Ex. 3. By the General Rule. (*See* Rule 2, and Table, Prob. X. Superficies.) $9^2 \times 4{\cdot}828427 =$ 391·102587 sq: inches, the area of the greater end. *Again.*—By Rule 2, and Table, Prob. X. Superficies. $5^2 \times 4{\cdot}828427 =$ 120·710675 sq. inches, the area of the less end.

Then, by the Rule, $\sqrt{391{\cdot}102587 \times 120{\cdot}710675} = \sqrt{47210{\cdot}257271016225} =$ 217·279215 sq. inches, the mean area, and $217{\cdot}279215 + 391{\cdot}102587 + 120{\cdot}710675 =$ 729·092477 square inches, the sum. Then, $\dfrac{729{\cdot}092477 \times (10{\cdot}5 \text{ ft.} \div 3)}{144} =$ 17·7209977 sol. feet, the content of the frustrum of the octagonal pyramid $=$ *Answer.*

To find the Superficies.

9 in. × 8 = 72 inches, the perimeter of the greater end,

and 5 in. × 8 = 40 inches, the perimeter of the less end;

The sum = 112 inches; which ÷ by 2 = 56 inches, the half sum.

Then, by the Note. (*See also* Rule 2, and Table, Prob. X. Superficies.) $(9 \text{ in.} - 5 \text{ in.}) \times 1{\cdot}2071068 =$ 4·8284272 inches $=$ PG. Now, *$\sqrt{PG^2 + GO^2} = \sqrt{4{\cdot}8284272^2 + 126^2} = \sqrt{15899{\cdot}31370922569984} =$ 126·0924807 inches, the slant height PO. And, by the Note. $126{\cdot}0924807 \times 56 =$ 7061·1789192 sq. inches, the surface of the sides.

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* Euclid, 47 p. 1. b.

Then, $\dfrac{7061{\cdot}1789192 + 391{\cdot}102587 + 120{\cdot}710675}{144}$ $= 52{\cdot}5902234$ sq. feet, the surface of the whole frustum = *Answer.*

Ex. 4. By Rule 2. $(AB \times DE) + AB^2 + DE^2 = (25 \text{ in.} \times 9 \text{ in.}) + 25^2 + 9^2 = 931$ square inches. Then, $\dfrac{931 \times (20 \text{ ft.} \div 3)}{144} = 43{\cdot}1018$ solid feet, the content of the frustum = *Answer.*

Ex. 5. By the General Rule. 32 in. × 20 in. = 640 sq. inches, the area of the greater end; and 10 in. × 6 in. = 60 sq. inches, the area of the less end.

Then, by the Rule. $\sqrt{640 \times 60} = \sqrt{38400} = 195{\cdot}959179$ sq. inches, the mean area; and $195{\cdot}959179 + 640 + 60 = 895{\cdot}959179$ sq. inches, the sum. Then, $\dfrac{895{\cdot}959179 \times (18 \text{ ft.} \div 3)}{144} = 37{\cdot}331632$ solid feet, the content of the frustum = *Answer.*

Ex. 6. By Rule 3. $(7{\cdot}5^2 + 4{\cdot}25^2) + (7{\cdot}5 \times 4{\cdot}25) = 106{\cdot}1875$ inches, the sum.

Then, 106·1875 × (4·828427, tab. area, ÷ 3) × 12·75 ft. = 2179·053113671875; which ÷ by 144 = 15·132313289388 solid feet, the solidity of one pillar; hence 15·132313289388 × 4 = 60·529253157552 feet, the solidity of the four pillars.

Then, as 1 sol. ft. : 60·529253157552 sol. ft. :: 12*s*. 10*d*. : £38. 16*s*. 9½*d*. ·02, the expense of the four pillars = *Answer.*

Ex. 7. (*See the Diagram and Demonstration to* Prob. VII. Solids.) *AG — CO = (AE) : CO :: GO : OV; *viz.* 2½ ft. — 1½ ft. : 1½ ft. :: 8 ft. : 12 ft.; ∴ GO + OV = 8 + 12 = 20 ft. = GV, the perpendicular height of the pyramid ABVA; and

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* Euclid, 4 p. vi. b.

$AB^2 = 5^2 = 25$ sq. feet, the area of the base of the pyramid. Then, by Rule, Prob. V. Solids. $25 \times (20 \text{ ft.} \div 3) = 166\frac{2}{3}$ solid feet, the content of the pyramid = *Answer.*

Ex. 8. By Rule 3: $(13^2 + 8^2) + (13 \times 8) = 337$, the sum.

Then, $387 \times (2{\cdot}598076$, tab. area, $\div 3) \times 24 = 7004{\cdot}4102$, the solidity of the frustum of the hexagonal pyramid = *Answer.*

To find the Superficies.

By Rule 2, and Table, Prob. X. Superficies. $13^2 \times 2{\cdot}598076 = 439{\cdot}074844$, the area of the greater end. *Again.*—By Rule 2, and Table, Prob. X. Superficies. $8^2 \times 2{\cdot}598076 = 166{\cdot}276864$, the area of the less end. Then, $13 \times 6 = 78$, the perimeter of the greater end.
And $8 \times 6 = 48$, the perimeter of the less end.

The sum = 126; which ÷ by 2 = 63, the half sum.

Then, by the Note. (*See also* Rule 2, and Table, Prob. X. Superficies.) $(13 - 8) \times {\cdot}8660254 = 4{\cdot}330127 = PG$. Now, *$\sqrt{PG^2 + GO^2} = \sqrt{4{\cdot}330127^2 + 24^2} = \sqrt{594{\cdot}749999836129} = 24{\cdot}387496$, the slant height PO. And, by the Note. $24{\cdot}387496 \times 63 = 1536{\cdot}412248$, the surface of the sides. Then, $1536{\cdot}412248 + 439{\cdot}074844 + 166{\cdot}276864 = 2141{\cdot}763956$, the whole surface = *Answer.*

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* Euclid, 47 p. i. b.

PROBLEM VIII.

To find the Solidity of the Frustum of a Cone.

Ex. 2. By Rule 1. (*See* Rule 2, Prob. XIII. Superficies.)

$36^2 \times {\cdot}7854 = \overset{\text{Square Inches.}}{1017{\cdot}8784}$, the area of the greater end.

And $9^2 \times {\cdot}7854 = 63{\cdot}6174$, the area of the less end. Then,

$$\left.\begin{array}{l}\sqrt{1017{\cdot}8784 \times 63{\cdot}6174}\\ = \sqrt{64754{\cdot}77732416}\end{array}\right\} = 254{\cdot}4696, \text{ the mean area.}$$

$$1335{\cdot}9654, \text{ the sum.}$$

Then, $\dfrac{1335{\cdot}9654 \times (24 \text{ ft.} \div 3)}{144} = 74{\cdot}2203$ solid feet, the content of the frustum = *Answer.*

Ex. 3. By Rule 2. (*See* Rule 2, Prob. XI. Superficies.) 136 in. × ·318309 = 43·290024 inches, the diameter of the greater end.

Again:—By Rule 2, Prob. XI. Superficies. 32 in. × ·318309 = 10·185888 inches, the diameter of the less end.

Then, by the Rule. $(43{\cdot}290024 \times 10{\cdot}185888) + 43{\cdot}290024^2 + 10{\cdot}185888^2 = 2418{\cdot}725828250432$ sq. inches; which ÷ by 144 = 16·796707140628 sq. feet And, 16·796707140628 × 21 ft. × (·7854 ÷ 3) = 92·344936517744 6184 solid feet = *Answer.*

PROBLEM IX.

To find the Solidity of a Wedge.

Ex. 2. $(2AE + ID) \times FE \times aC \times \frac{1}{6} = \overline{(32 \times 2 + 21)} \times 4\frac{1}{2} \times 14 \times \frac{1}{6} = 892\frac{1}{2}$ cubic inches, the solidity of the wedge AEDIBF = *Answer.*

Ex. 3. $(2\ \mathrm{AE} + \mathrm{ID}) \times \mathrm{FE} \times a\mathrm{C} \times \frac{1}{6} = \overline{(5\frac{1}{3} \times 2}$ $\overline{+\ 3\frac{1}{2})} \times \frac{3}{4} \times 2\frac{1}{3} \times \frac{1}{6} = 4\frac{19}{144} = 4{\cdot}131944$ cubic feet, the solidity of the wedge AEDIBF = *Answer.*

PROBLEM X.

To find the Solidity of a Prismoid.

$$\left.\begin{array}{l}\text{Ex. 2.}\ (\mathrm{AB} + \frac{1}{2}\ \mathrm{GH}) \times \mathrm{AC} = (13 + \\ \overline{12 \div 2}) \times 13\ .\ .\ .\ .\ .\ .\ .\ .\end{array}\right\} = 247$$

$$(\mathrm{GH} + \tfrac{1}{2}\ \mathrm{AB}) \times \mathrm{GE} = (12 + \overline{13 \div 2}) \times 5 = \ \ 92\tfrac{1}{2}$$

$$\text{The sum} = 339\tfrac{1}{2}$$

Then, $339\frac{1}{2} \times (20 \div 3) = 2263\frac{1}{3}$, the solidity of the prismoid ABCDEFGH = *Answer.*

$$\left.\begin{array}{l}\text{Ex. 3.}\ (\mathrm{AB} + \frac{1}{2}\,\mathrm{GH}) \times \\ \mathrm{AC} = (81{\cdot}5 + \overline{41 \div 2}) \times 55\end{array}\right\} = 5610{\cdot}000 \text{ sq. inches,}$$

$$\left.\begin{array}{l}(\mathrm{GH} + \frac{1}{2}\mathrm{AB}) \times \mathrm{GE} = (41 \\ \quad + \overline{81{\cdot}5 \div 2}) \times 29{\cdot}5\ \ .\end{array}\right\} = 2411{\cdot}625 \text{ sq. inches,}$$

$$\text{The sum} = 8021{\cdot}625 \text{ sq. inches.}$$

$$\text{Then, } \frac{8021{\cdot}625 \times (47{\cdot}25 \text{ in.} \div 3)}{1728} = 73{\cdot}11376953125$$

solid feet, the content of the prismoid ABCDEFGH = *Answer.*

$$\left.\begin{array}{l}\text{Ex. 4.}\ (\mathrm{AB} + \frac{1}{2}\,\mathrm{GH}) \times \\ \mathrm{AC} = (3648 + \overline{3600 \div 2}) \\ \quad \times 240\ .\ .\ .\ .\ .\ .\ .\end{array}\right\} = 1307520 \text{ sq. inches,}$$

$$\left.\begin{array}{l}(\mathrm{GH} + \frac{1}{2}\mathrm{AB}) \times \mathrm{GE} = (3600 \\ \quad + \overline{3648 \div 2}) \times 192\ .\end{array}\right\} = 1041408 \text{ sq. inches,}$$

$$\text{The sum} = 2348928 \text{ sq. inches.}$$

$$\text{Then, } \frac{2348928 \times (60 \text{ in.} \div 3)}{282} = 166590{\cdot}63829$$

gallons of water (old measure), and* $\overline{166590{\cdot}68829 \times 235} \div 231 = 169475{\cdot}32466$ imperial gallons, the content of the canal ABCDEFGH = *Answer.*

PROBLEM XI.

To find the Solidity of a Cylindroid; or the frustum of an elliptical Cone.

$$\left.\begin{array}{l}\text{Ex. 2. By Rule 1. } (AB + \tfrac{1}{2}\,CD) \times \\ GH = (13 + \overline{10 \div 2}) \times 8 \quad . \;\; . \;\; . \;\; .\end{array}\right\} = 144{\cdot}0$$

$$\left.\begin{array}{l}(CD + \tfrac{1}{2}\,AB) \times mP = (10 + \overline{13 \div 2}) \\ \times\, 5{\cdot}2 \quad . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; .\end{array}\right\} = \underline{\;\;85{\cdot}8\;\;}$$

$$\text{The sum} = 229{\cdot}8$$

Then, $229{\cdot}8 \times ({\cdot}7854 \div 3) \times 120 = 7219{\cdot}3968$, the solidity of the cylindroid ACDBA = *Answer.*

Or thus, by Rule 2. (*See* Rule, Prob. XXI. Superfi.)

$$\left.\begin{array}{l}AB \times GH \times {\cdot}7854 \\ = 13 \times 8 \times {\cdot}7854\end{array}\right\} = 81{\cdot}68160 \left\{\begin{array}{l}\text{the area of the} \\ \text{greater end} \\ \text{AGBH.}\end{array}\right.$$

$$\left.\begin{array}{l}CD \times mP \times {\cdot}7854 \\ = 10 \times 5{\cdot}2 \times \\ {\cdot}7854 \quad . \;\; . \;\; . \;\; .\end{array}\right\} = 40{\cdot}84080 \left\{\begin{array}{l}\text{the area of} \\ \text{the less end} \\ CmDP.\end{array}\right.$$

$$\left.\begin{array}{l}\tfrac{1}{2}\,(AB + CD) \times \tfrac{1}{2} \\ (GH + mP) \times {\cdot}7854 \\ \times\, 4 = \tfrac{1}{2}\,(13 + 10) \\ \times\, \tfrac{1}{2}\,(8 + 5{\cdot}2) \times \\ {\cdot}7854 \times 4 \quad . \;\; .\end{array}\right\} = \underline{238{\cdot}44744} \left\{\begin{array}{l}\text{four times the} \\ \text{area of the} \\ \text{middle sec-} \\ \text{tion.}\end{array}\right.$$

$$\text{The sum} = 360{\cdot}96984.$$

Then, $360{\cdot}96984 \times (120 \div 6) = 7219{\cdot}3968$, the solidity of the cylindroid ACDBA as before = *Answer.*

$$\left.\begin{array}{l}\text{Ex. 3. By Rule 1. } (AB + \tfrac{1}{2}\,CD) \\ \times\, GH = (12 + \overline{10 \div 2}) \times 7 \quad . \;\; .\end{array}\right\} = 119 \text{ sq. in.}$$

$$\left.\begin{array}{l}(CD + \tfrac{1}{2}\,AB) \times mP = (10 + \overline{12 \div 2}) \\ \times\, 6 \quad . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; . \;\; .\end{array}\right\} = \underline{\;96 \text{ sq. in.}}$$

$$\text{The sum} = 215 \text{ sq. in.}$$

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* See Rule 2, ale and beer measure, in this Key, under the Article *Gauging*.

Then, 215 × (·7854 ÷ 3) × 4 ft. = 225·148 cubic feet, the solidity.

And $\frac{225·148 \times 1728}{282} = 1379·63029$ ale gallons; and by Rule 2, ale and beer measure, *of this Key*, $\overline{1379·63029 \times 235} \div 231 = 1403·51999$ imp. gallons, the content of the elliptical bath ACDBA = *Answer*.

PROBLEM XII.

To find the Solidity of a Sphere or Globe.

Ex. 2. By Rule 1. $AB^3 \times ·5236 = 7970^3 \times ·5236 = 265078559622·8$ cubic miles, the solidity of the earth = *Answer.*

To find the Superficies.

By Rule 2. (*See* Rule 1. Prob. XI. Superficies.) AB × 3·1416 = 7970 × 3·1416 = 25038·552 miles, the circumference. Then, by the Rule. AB × 25038·552 = 7970 × 25038·552 = 199557259·44 sq. miles, the convex superficies of the earth = *Answer.*

Ex. 3, By Rule 1. (*See* Rule 2, Prob. XI. Superficies.) 1 × ·318309 = ·318309, the diameter AB. Then, by the Rule. $AB^3 \times ·5236 = ·318309^3 \times ·5236 = ·0168867623887553265444$, the solidity of the sphere = *Answer.*

To find the Superficies.

By Rule 2. The diameter AB × 1 = ·318309 × 1 = ·318309, the superficies = *Answer.*

To find the Superficies and Solidity.

Ex. 4. By Rule 2. (*See* Rule 1. Prob. XI. Superficies.) AB × 3·1416 = 30 × 3·1416 = 94·248, the circumference. Then, by the Rule. AB × 94·248 = 30 × 94·248 = 2827·44, the convex surface of the globe = 1*st Answer.*

Now, the surface as above = 2827·44; hence, 2827·44 × (30 ÷ 6) = 14137·2, the solidity of the globe = 2*nd Answer.*

To find the Superficies and Solidity.

Ex. 5. By Rule 2. (*See* Rule 2, Prob. XI. Superficies.) 50·3 × ·318309 = 16·0109427, the diameter AB. Then, by the Rule. 16·0109427 × 50·3 = 805·35041781, the convex surface of the globe = 1*st Answer.*

Now, the surface as above = 805·35041781; hence, 805·35041781 × (16·0109427 ÷ 6) = 2149·069898829494914, the solidity of the globe = 2*nd Answer.*

Ex. 6. *First, for the Globe.* If the superficies of a globe be divided by 3·1416, the quotient will be the square of the diameter, the square root of which will be the diameter; hence, the solidity is found as follows:

The surface of the globe (by the Question,) is 3·1416, which ÷ by 3·1416 = 1, the square of the diameter. Then, $\sqrt{1} = 1$, the diameter. Now, by Rule 1, $1^3 \times$ ·5236 = ·5236, the solidity of the globe = 1*st Answer.*

Second, for the Cube. The cube has six equal surfaces; therefore the whole surface divided by 6 will give the square of the side of the cube; ∴ the square root will be the side. Hence, the solidity is found as follows:

The surface of the cube (by the Question) is 3·1416, which ÷ by 6 = ·5236, the square of one of its sides; hence, $\sqrt{\cdot 5236} = \cdot 723602$, one of its sides. Then, by Rule, Prob. I. Solids. $\cdot 723602^3 = \cdot 378877901846443208$, the solidity of the cube = 2*nd Answer.*

Third, for the Cylinder. The diameter and depth of the Cylinder are here supposed equal. The superficies of a cylinder, whose diameter and depth are each an unit, will be found to be 4·7124; and the superficies of similar cylinders are to each other as the squares of their diameters; therefore it will be as 4·7124 is to the square of 1, so is the superficies of the given cylinder to the square of its diameter; ∴ the square root of which will be the diameter, and likewise the depth. Hence, the solidity of the cylinder is found as follows:

As $4{\cdot}7124 : 1^2 :: 3{\cdot}1416 : {\cdot}6'$, the square of its diameter; hence $\sqrt{{\cdot}6'} = {\cdot}81649658$, the diameter and likewise the depth. Then, by Rule, Prob. IV. Solids. $\cdot 81649658^2 \times {\cdot}7854 = {\cdot}5235999881014235256$, the area of the base of the cylinder, which × by ·81649658, (the depth) $= {\cdot}4275176083164853001783942448$, the solid content of the cylinder = *3rd Answer.*

PROBLEM XIII.

To find the Solidity of the Segment of a Sphere.

Ex. 2. By Rule 2. $(3\,AO^2 + OD^2) \times OD \times {\cdot}5236 = (\overline{10^2 \times 3} + 9^2) \times 9 \times {\cdot}5236 = 1795{\cdot}4244$, the solidity of the segment ABDO = *Answer.*

Ex. 3. By Rule 1. $(3\,ED - 2\,OD) \times OD^2 \times {\cdot}5236 = (\overline{20 \times 3} - \overline{5 \times 2}) \times 5^2 \times {\cdot}5236 = 654{\cdot}5$ feet, the solidity of the segment ABDO = *Answer.*

Ex. 4. By Rule 1. $(3\,ED - 2\,OD) \times OD^2 \times {\cdot}5236 = (\overline{21 \times 3} - \overline{4{\cdot}5 \times 2}) \times 4{\cdot}5^2 \times {\cdot}5236 = 572{\cdot}5566$, the solidity of the segment ABDO = *Answer.*

Ex. 5. By Rule 2. $(3\,AO^2 + OD^2) \times OD \times {\cdot}5236 = (\overline{14^2 \times 3} + 6{\cdot}5^2) \times 6{\cdot}5 \times {\cdot}5236 = 2144{\cdot}99285$, the solidity of the segment ABDO = *Answer.*

PROBLEM XIV.

To find the Solidity of the Frustum or Zone of a Sphere.

Ex. 2. By Rule 2. $(EF^2 - \frac{1}{3}\ mO^2) \times mO \times \cdot 7854 = (22^2 - \overline{14^2 \div 3}) \times 14 \times \cdot 7854 = 4603 \cdot 4912$ cubic inches, the solidity of the zone ABDC = *Answer*.

Ex. 3. By Rule 1. $EG^2 + CO^2 + \frac{1}{3}\ GO^2 = 6^2 + 5^2 + \overline{2^2 \div 3} = 62 \cdot 3'$, the sum.

Then, $62 \cdot 3' \times 2 \times 1 \cdot 5708 = 195 \cdot 8264$, the solidity of the zone EFDC = *Answer*.

Ex. 4. By Rule 2. $(AB^2 + \frac{2}{3}\ mO^2) \times mO \times \cdot 7854 = (3^2 + \overline{4^2 \times \frac{2}{3}}) \times 4$ ft. $\times \cdot 7854 = 61 \cdot 7848$ feet, the solidity of the zone ABDC = *Answer*.

Ex. 5. By Rule 2. $(EF^2 - \frac{1}{3}\ mO^2) \times mO \times \cdot 7854 = (5^2 - \overline{4^2 \div 3}) \times 4$ ft. $\times \cdot 7854 = 61 \cdot 7848$ feet, the solidity of the zone ABDC = *Answer*.

PROBLEM XV.

To find the convex surface of any Segment, or Zone, of a Sphere.

Ex. 1. By Rule 1, Prob. XI. Superficies. 7970 × 3·1416 = 25038·552 miles, the circumference of the earth.

Then, by the Rule. 25038·552 × 252·361283 = 6318761·107182216 sq. miles, the surface of the frigid zone = *Answer*.

Ex. 2. By Rule 1. Prob. XI. Superficies. 7970 × 3·1416 = 25038·552 miles, the circumference of the earth.

Then, by the Rule. 25038·552 × 2143·6235535

= 53673229·812734532 sq. miles, the surface of the temperate zone = *Answer.*

Ex. 3. By Rule 1, Prob. XI. Superficies. 7970 × 3·1416 = 25038·552 miles, the circumference of the earth.

Then, by the Rule. 25038·552 × 3178·030327 = 79573277·600166504 sq. miles, the surface of the torrid zone = *Answer.*

PROBLEM XVI.

To find the Solidity of a circular Spindle.

Ex. 2. *First.* $\frac{1}{2}$ CD = 36 ÷ 2 = 18 = OC. Then, by Note 8, Prob. XII. Superficies. $(AO^2 \div OC) + OC = (24^2 \div 18) + 18 = 50$, the diameter of the circle CF. Now, by the Rule. $\frac{1}{2}CF - OC = \overline{50 \div 2 - 18} = 7$, the *central distance* EO.

Second. By Rule to Ex. 1. Prob. XVII. Superficies. OC ÷ CF = 18 ÷ 50 = ·360, the tabular height; against which stands the *Area Seg.* ·254550: hence $·254550 \times 50^2 = 636·375$, the *area of the generating segment* ACB, which ÷ by 2 = 318·1875 = ACO, the area of half the generating segment.

Third. By the Rule. $\frac{1}{3}AO^3 - (EO \times ACO) = (24^3 \div 3) - (7 \times 318·1875) = 2380·6875$; which × by 12·5664 = 29916·6714, the solidity of the circular spindle ACBDA = *Answer.*

To find the Superficies.

First. By the Note. The central distance EO = 7, as above.

Then, *$\sqrt{AO^2 + OC^2} = \sqrt{24^2 + 18^2} = \sqrt{900} = 30$ = AC, the chord of half the arc ACB. Then, by Rule 2, Prob. XII. Superficies. $\frac{1}{3}\,(8\ \mathrm{AC} - \mathrm{AB}) = \frac{1}{3}\,(\overline{30 \times 8} - 48) = 64$, the length of the arc ACB.

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* Euclid, 47 p. i. b.

Second. By the Note. $(\mathrm{AB} \times \mathrm{EC}) - (\mathrm{EO} \times \mathrm{ACB}) \times 6{\cdot}2832 = (48 \times 25) - (7 \times 64) \times 6{\cdot}2832 = 4724{\cdot}9664$, the superficies of the circular spindle ACBDA = *Answer.*

PROBLEM XVII.

To find the Solidity of the middle Frustum, or Zone, of a circular Spindle.

Ex. 2. *First.* By the Rule. $\frac{1}{2}\,(\mathrm{DC} - \mathrm{PN}) = \frac{1}{2}\,(60 - 36) = 12$ inches $= \mathrm{OC}$, the height of the segment $m\mathrm{NC}m$. Then, by Note 8. Prob. XII. Superficies. $(m\mathrm{O}^2 \div \mathrm{OC}) + \mathrm{OC} = (40^2 \div 12) + 12 = 145{\cdot}3'$ inches, the diameter of the circle; which $\div$ by $2 = 72{\cdot}6'$ inches, the radius EC. Also, $\mathrm{EC} - \mathrm{GC} = 72{\cdot}6' - 30 = 42{\cdot}6'$ inches, the *central distance* EG.

Second. By Rule 2, Prob. XVI. Superficies. $\sqrt{m\mathrm{O}^2 + \mathrm{OC}^2} = \sqrt{40^2 + 12^2} = \sqrt{1744} = 41{\cdot}761226$ inches $= \mathrm{C}m$, the chord of half the arc $m\mathrm{CN}$. Then, $\sqrt{m\mathrm{N}^2 + \mathrm{OC}^2} = \sqrt{80^2 + 12^2} = \sqrt{6544} = 80{\cdot}894993$. Now, $(\overline{80{\cdot}894993 \times 2} + \mathrm{C}m) \times \mathrm{OC} \times \frac{4}{15} = (\overline{80{\cdot}894993 \times 2} + 41{\cdot}761226) \times 12 \times \frac{4}{15} = 651{\cdot}363878$ sq. inches, the area of the segment $m\mathrm{NC}m$; and $m\mathrm{N} \times \frac{1}{2}\,\mathrm{NP} = 80 \times (36 \div 2) = 1440$ sq. inches, the area of the rectangle $m\mathrm{N}num$. Then, $m\mathrm{NC}m + m\mathrm{N}num = 651{\cdot}363878 + 1440 = 2091{\cdot}363878$ sq. inches, the *generating surface* $um\mathrm{CN}nu$.

Third. *$\sqrt{\mathrm{AE}^2 - \mathrm{EG}^2} = \sqrt{72{\cdot}6'^2 - 42{\cdot}6'^2} = \sqrt{3460} = 58{\cdot}821764$ inches = AG, half the length of the spindle AB.

Fourth. $(\mathrm{AG}^2 - \frac{1}{3}\,m\mathrm{O}^2) \times m\mathrm{O} = (3460 - \overline{1600 \div 3}) \times 40 = 117066{\cdot}6'$. Then $\mathrm{EG} \times um\mathrm{CN}nu$

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* Euclid, 47 p. i. b.

$= 42\cdot6' \times 2091\cdot363878 = 89231\cdot525461$; and $(117066\cdot6' - 89231\cdot525461) \times 6\cdot2832 = 174893\cdot7592192560$ cubic inches, the solidity of the middle frustum *m*CN*n*PD*um*, which ÷ by 282 = 620·1906355292 ale gallons; and, by Rule 2, ale and beer measure *of this Key*, $620\cdot1906355292 \times 235 \div 231 = 630\cdot9298673132$ imperial gallons, the content of the frustum = *Answer.*

PROBLEM XVIII.

To find the Solidity of a Spheroid.

Ex. 2. $AB^2 \times CE \times \cdot5236 = 55^2 \times 33 \times .5236 = 52268\cdot37$ cubic inches, the solidity of the oblate spheroid ACBEA = *Answer.*

Ex. 3. $CE^2 \times AB \times \cdot5236 = 30^2 \times 50 \times \cdot5236 = 23562$, the solid content of the prolate spheroid ACBEA = *Answer.*

Ex. 4. $AB^2 \times CE \times \cdot5236 = 50^2 \times 30 \times \cdot5236 = 39270$, the solidity of the oblate spheroid ACBEA = *Answer.*

PROBLEM XIX.*

To find the Solidity of the Segment of a Spheroid, the Base of the Segment being parallel to the revolving Axis of the Spheroid.

Ex. 2. $(\overline{3\ AB - 2}\ mB) \times mB^2 \times \cdot5236 = (\overline{100 \times 3 - 10 \times 2}) \times 10^2 \times \cdot5236 = 14660\cdot8$, the lidity of the spherical segment *acBa*.

$AB^2 : CD^2 :: acBa : GHBG$; *viz.* $100^2 : 60^2 :: 14660\cdot8 : 5277\cdot888$, the solidity of the elliptical or spheroidal segment = *Answer.*

Ex. 3. $(3\ AB - 2\ mB) \times mB^2 \times \cdot5236 =$

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* See Figure 8 of this Key.

$\overline{(50 \times 3} - \overline{5 \times 2)} \times 5^2 \times \cdot5236 = 1832\cdot6$, the solidity of the spherical segment *acBa*.

$AB^2 : CD^2 :: acBa : GHBG$; *viz.* $50^2 : 30^2 :: 1832\cdot6 : 659\cdot736$, the solidity of the elliptical or spheroidal segment = *Answer.*

Ex. 4. $(3\,CD - 2\,PC) \times PC^2 \times \cdot5236 = \overline{(30 \times 3} - \overline{6 \times 2)} \times 6^2 \times \cdot5236 = 1470\cdot2688$, the solidity of the spherical segment *reCr*.

$CD^2 : AB^2 :: reCr : NVCN$; *viz.* $30^2 : 50^2 :: 1470\cdot2688 : 4084\cdot08$, the solidity of the elliptical segment = *Answer.*

Ex. 5. $(3\,AB - 2\,mB) \times mB^2 \times \cdot5236 = \overline{(10 \times 3} - \overline{1 \times 2)} \times 1^2 \times \cdot5236 = 14\cdot6608$, the solidity of the spherical segment *acBa*.

$AB^2 : CD^2 :: acBa : GHBG$; *viz.* $10^2 : 6^2 :: 14\cdot6608 : 5\cdot277888$, the solidity of the spheroidal segment = *Answer.*

Ex. 6. $(3\,AB - 2\,mB) \times mB^2 \times \cdot5236 = \overline{(36\tfrac{2}{3} \times 3} - \overline{6\tfrac{1}{2} \times 2)} \times \overline{6\tfrac{1}{2}}|^2 \times \cdot5236 = 2145\cdot8437$, the solidity of the spherical segment *acBa*.

$AB^2 : CD^2 :: acBa : GHBG$; *viz.* $\overline{36\tfrac{2}{3}}|^2 : 18^2 :: 2145\cdot8437 : 517\cdot1306$, the solidity of the spheroidal segment = *Answer.*

PROBLEM XX.

To find the Solidity of the middle Zone of a Spheroid, the Diameter of the Ends being perpendicular to the fixed Axis, the middle Diameter, and that of either End being given, together with the Length of the Zone.

Ex. 2. $(2\,EF^2 + mn^2) \times uO \times \cdot2618 = \overline{(100^2 \times 2} + 80^2) \times 36 \times \cdot2618 = 248814\cdot72$, the solidity of the middle zone of an oblate spheroid TE*mn*FPT = *Answer.*

Ex. 3. $(2\,EF^2 + mn^2) \times uO \times \cdot2618 = \overline{(32^2 \times 2 + 24^2)} \times 40 \times \cdot2618 = 27478\cdot528$ cubic inches, the solidity of the spheroidal cask TE*mn*FPT; which ÷ by $282 = 97\cdot4415$ ale gallons; and, by Rule 2, ale and beer measure *of this Key*, $\overline{97\cdot4415 \times 235} \div 231 = 99\cdot1287$ imperial gallons, the content of the cask = *Answer.*

Ex. 4. $(2\,EF^2 + TP^2) \times uO \times \cdot2618 = \overline{(16^2 \times 2 + 12^2)} \times 20 \times \cdot2618 = 3434\cdot816$ cubic inches, the solidity of the spheroidal cask TE*mn*FPT; which ÷ by $231 = 14\cdot8693$ wine gallons; and [*]$14\cdot8693 - (14\cdot8693 \div 6) = 12\cdot3911$ imperial gallons, the content of the cask = *Answer.*

PROBLEM XXI.

To find the Solidity of a Parabolic Conoid.

Ex. 2. $AB^2 \times \cdot3927 \times CD = 48^2 \times \cdot3927 \times 84 = 76001\cdot5872$, the solidity of the parabolic conoid ACBDA = *Answer.*

Ex. 3. $AB^2 \times \cdot3927 \times CD = 40^2 \times \cdot3927 \times 30 = 18849\cdot6$, the solidity of the paraboloid ACBDA = *Answer.*

Ex. 4. $AB^2 \times \cdot3927 \times CD = 32^2 \times \cdot3927 \times 22\cdot85 = 9188\cdot55168$, the solidity of the paraboloid ACBDA = *Answer.*

PROBLEM XXII.

To find the Solidity of the Frustum of a Parabolic Conoid.

Ex. 2. *(See Fig. 2.)* $(AB^2 + mn^2) \times CO \times \cdot3927$

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[*] See the Rule, in wine measure, *of this Key*, under the Article *Gauging*.

$= (30^2 + 24^2) \times 9 \times {\cdot}3927 = 5216{\cdot}6268$, the solidity of the frustum of the parabolic conoid ACB*n*O*m*A = *Answer.*

Ex. 3. (*See Fig.* 2.) $(AB^2 + mn^2) \times CO \times {\cdot}3927 = (60^2 + 48^2) \times 18 \times {\cdot}3927 = 41733{\cdot}0144$, the solidity of the frustum of the paraboloid ACB*n*O*m*A = *Answer.*

Ex. 4. (*See Fig.* 1.) $(BAB^2 + bab^2) \times Aa \times {\cdot}3927 = (32^2 + 24^2) \times 20 \times {\cdot}3927 = 12566{\cdot}4$ cubic inches = BAB*bab*B, the solid content of half the cask, which × by 2 = 25132·8 cubic inches, the solid content of the whole cask *b*B*bab*B*bb*.

Then, $25132{\cdot}8 \div 282 = 89{\cdot}1234$ ale gallons; and, by Rule 2, ale and beer measure, *of this Key,* $\overline{89{\cdot}1234 \times 235} \div 231 = 90{\cdot}6666$ imperial gallons, the content of the cask = *Answer.*

Ex. 5. (*See Fig.* 1.) $(BAB^2 + bab^2) \times Aa \times {\cdot}3927 = (16^2 + 12^2) \times 10 \times {\cdot}3927 = 1570{\cdot}8$ cubic inches = BAB*bab*B, the solid content of half the cask; which × by 2 = 3141·6 cubic inches, the solid content of the whole cask *b*B*bab*B*bb*. Then, $3141{\cdot}6 \div 231 = 13{\cdot}6$ wine gallons; and, by the Rule, wine measure, *of this Key,* $13{\cdot}6 - (13{\cdot}6 \div 6) = 11{\cdot}4$ imperial gallons, the content of the cask = *Answer.*

PROBLEM XXIII.

To find the Solidity of a Parabolic Spindle.

Ex. 2. $CD^2 \times {\cdot}7854 \times AB \times \frac{8}{15} = 34^2 \times {\cdot}7854 \times 60 \times \frac{8}{15} = 29053{\cdot}5168$, the solidity of the parabolic spindle ABCD = *Answer.*

Ex. 3. $CD^2 \times {\cdot}41888 \times AB = 3^2 \times {\cdot}41888 \times 9$

= 33·92928, the solidity of the parabolic spindle ABCD = *Answer*.

Ex. 4. $CD^2 \times \cdot7854 \times AB \times \frac{8}{15} = 16^2 \times \cdot7854 \times 40 \times \frac{8}{15} = 4289\cdot3312$, the solidity of the parabolic spindle ABCD = *Answer*.

PROBLEM XXIV.

To find the Solidity of the middle Frustum of a Parabolic Spindle.

Ex. 2. $(2\,CD^2 + EF^2) - (\cdot4 \times \overline{CD - EF}|^2) \times AB \times \cdot2618 = (\overline{32^2 \times 2} + 24^2) - (\cdot4 \times \overline{32 - 24}|^2) \times 40 \times \cdot2618 = 27210\cdot4448$ cubic inches, the solidity of the cask GEFHG; which ÷ by 282 = 96·4909 ale gallons; and, by Rule 2, ale and beer measure, *of this Key*, $\overline{96\cdot4909 \times 235} \div 231 = 98\cdot1617$ imperial gallons, the content of the cask = *Answer*.

Ex. 3. $(2\,CD^2 + EF^2) - (\cdot4 \times \overline{CD - EF}|^2) \times AB \times \cdot2618 = (\overline{36^2 \times 2} + 20^2) - (\cdot4 \times \overline{36 - 20}|^2) \times 36 \times \cdot2618 = 27233\cdot90208$ cubic inches, the solidity of the cask GEFHG, which ÷ by 231 = 117·89568 wine gallons; and, by the Rule, wine measure, *of this Key*, $117\cdot89568 - (117\cdot89568 \div 6) = 98\cdot2464$ imperial gallons, the content of the cask = *Answer*.

PROBLEM XXV.*

To find the Solidity of an hyperbolic Conoid.

Ex. 2. $\dfrac{(2\,mV + 3\,VE) \times Am^2 \times mV \times \cdot5236}{mV + VE}$

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* See *Keith's Mensuration*, the note to Definition 23, Solids.

$= \frac{(\overline{10\times 2}+\overline{30\times 3})\times 12^2\times 10\times \cdot 5236}{10+30} =$ $\frac{82938\cdot 24}{40} = 2073\cdot 456$, the solidity of the hyperboloid $AmBVA$ = *Answer.*

Ex. 3. $\frac{(2\,mV+3\,VE)\times Am^2\times mV\times \cdot 5236}{mV+VE}$ $= \frac{(\overline{50\times 2}+\overline{122\cdot 449\times 3})\times 52^2\times 50\times \cdot 5236}{50+122\cdot 449}$ $= \frac{33083830\cdot 61984}{172\cdot 449} = 191847\cdot 0424$, the solidity of the hyperboloid $AmBVA$ = *Answer.*

Ex. 4. Here CD $= 6\sqrt{7}$, or, which is the same, $\sqrt{6\times 6\times 7}=\sqrt{252}=15\cdot 874507$, which ÷ by 2 $= 7\cdot 937253$, the ordinate CO.

[To get the transverse axis, *see* the demonstration to this Problem.]

$\frac{4\,CO^2-Am^2}{2\,Am^2-4CO^2}\times mV = \frac{(7\cdot 937253^2\times 4)-12^2}{(12^2\times 2)-(7\cdot 937253^2\times 4)}$ $\times 10 = \frac{107\cdot 999940744036}{36\cdot 000059255964}\times 10 = 3\times 10 = 30$, the transverse axis VE.

Then, by the Rule. $\frac{(2mV+3VE)\times Am^2\times mV\times \cdot 5236}{mV+VE}$ $= \frac{(\overline{10\times 2}+\overline{30\times 3})\times 12^2\times 10\times \cdot 5236}{10+30} =$ $\frac{82938\cdot 24}{40} = 2073\cdot 456$, the solid content of the parabolic conoid $AmBVA$ = *Answer.*

Ex. 5. CD $= 35\cdot 49648$; which ÷ by $2 = 17\cdot 74824$, the ordinate CO.

Now, to get the transverse axis, *see* the demonstration to this Problem.

$\frac{4CO^2-Am^2}{2Am^2-4CO^2}\times mV = \frac{(17\cdot 74824^2\times 4)-30^2}{(30^2\times 2)-(17\cdot 74824^2\times 4)}$

$\times\ 50 = \frac{360{\cdot}0000923904}{539{\cdot}9999076096} \times 50 = {\cdot}\dot{6}' \times 50 = 33\frac{1}{3}$, the transverse diameter VE. Then, by the Rule. $\frac{(2\,mV + 3\,VE) \times Am^2 \times mV \times {\cdot}5236}{mV + VE}$ $= \frac{(\overline{50 \times 2} + \overline{33\frac{1}{3} \times 3}) \times 30^2 \times 50 \times {\cdot}5236}{50 + 33\frac{1}{3}} = \frac{4712400}{83\frac{1}{3}}$ $= 56548{\cdot}8$, the solid content of the parabolic conoid AmBVA = *Answer.*

PROBLEM XXVI.

To find the Solidity of the Frustum of an hyperbolic Conoid.

Ex. 2. $(4\,CD^2 + AB^2 + EH^2) \times mn \times {\cdot}1309 = (\overline{28{\cdot}17^2 \times 4} + 32^2 + 24^2) \times 20 \times {\cdot}1309 =$ 12498·8440808 cubic inches, the solidity of the frustum of the hyperboloid ACE*n*HDBA = *Answer.*

Ex. 3. $(4\,CD^2 + AB^2 + EH^2) \times mn \times {\cdot}1309 = (\overline{8{\cdot}5^2 \times 4} + 10^2 + 6^2) \times 12 \times {\cdot}1309 = 667{\cdot}59$, the solidity of the frustum of the hyperbolic conoid ACE*n*HDBA = *Answer.*

PROBLEM XXVII.

To find the Solidity of the middle Frustum of any Spindle, formed by the Revolution of a conic Section about its Axis.

Ex. 1. *See* the Figure to Prob. XXIV. Solids. $(CD^2 + EF^2 + 4\,IK^2) \times AB \times {\cdot}1309 = (32^2 + 24^2 + \overline{30{\cdot}157568^2 \times 4}) \times 40 \times {\cdot}1309 =$ 27425·726242337325056, the solidity of the middle frustum GEFHG = *Answer.*

Ex. 2. *See* the Figure to Prob. XXIV. Solids. $(CD^2$

$+ EF^2 + 4 IK^2) \times AB \times {\cdot}1309 = (16^2 + 12^2 + 14{\cdot}5^2 \times 4) \times 20 \times {\cdot}1309 = 3248{\cdot}938$ cubic inches, the solidity of the middle frustum GEFHG = *Answer.*

PROBLEM XXVIII.

To find the Solidity of a Cylindrical Ring.

Ex. 2. $(AC + CD) \times AC^2 \times 2{\cdot}4674 = (6{\cdot}5 + 25) \times 6{\cdot}5^2 \times 2{\cdot}4674 = 3283{\cdot}800975$ cubic inches, the solidity of the ring *mOnV* = *Answer.*

Ex. 3. By the Note. $(AC + CD) \times AC \times 9{\cdot}8696 = (4 + 18) \times 4 \times 9{\cdot}8696 = 868{\cdot}5248$ sq. inches, the superficies of the cylindrical ring *mOnV* = 1*st Answer.*

Then, as 1 sq. in. : 868·5248 sq. in. :: 1*d.* : £3. 12*s.* 4½*d.* ·0992, the expense of gilding the ring = 2*nd Answer.*

OF THE FIVE REGULAR BODIES.

PROBLEM XXIX.

To find the Solidity and Superficies of any regular Body.

Ex. 2. By the Rule, and Table. $AB^3 \times {\cdot}1178511 = 3^3 \times {\cdot}1178511 = 3{\cdot}1819797$, the solidity of the tetraedron = *Answer.*

To find the Superficies.

By the Rule, and Table. $AB^2 \times 1{\cdot}7320508 = 3^2$

$\times\ 1{\cdot}7320508 = 15{\cdot}5884572$, the superficies of the tetraedron = *Answer.*

Ex. 3. By the Rule, and Table. $AB^3 \times {\cdot}4714045 = 12^3 \times {\cdot}4714045 = 814{\cdot}586976$ cubic inches, the solidity of the octaedron = *Answer.*

To find the Superficies.

By the Rule, and Table. $AB^2 \times 3{\cdot}4641016 = 12^2 \times 3{\cdot}4641016 = 498{\cdot}8306304$ sq. inches, the superficies of the octaedron = *Answer.*

Ex. 4. By the Rule, and Table. $AB^3 \times 2{\cdot}1816950 = 12^3 \times 2{\cdot}1816950 = 3769{\cdot}96896$ cubic inches, the solidity of the icosaedron = *Answer.*

To find the Superficies.

By the Rule, and Table. $AB^2 \times 8{\cdot}6602540 = 12^2 \times 8{\cdot}6602540 = 1247{\cdot}076576$ sq. inches, the superficies of the icosaedron = *Answer.*

Ex. 5. By the Rule, and Table. $AB^3 \times 7{\cdot}6631189 = 12^3 \times 7{\cdot}6631189 = 13241{\cdot}8694592$ cubic inches, the solidity of the dodecaedron = *Answer.*

To find the Superficies.

By the Rule, and Table. $AB^2 \times 20{\cdot}6457288 = 12^2 \times 20{\cdot}6457288 = 2972{\cdot}9849472$ sq. inches, the superficies of the dodecaedron = *Answer.*

Ex. 6. By the Rule, and Table. $12^3 \times 1 = 1728$ cubic inches, the solidity of the hexaedron = *Answer.*

To find the Superficies.

By the Rule, and Table. $12^2 \times 6 = 864$ sq. inches, the superficies of the hexaedron = *Answer.*

OF THE SPECIFIC GRAVITY OF SOLIDS.

PROBLEM XXX.

To find the specific Gravity of a Body.

CASE I.

When the Body is heavier than Water.

Ex. 2. 108·5 lbs. — 72·3 lbs. = 36·2 pounds weight lost in water.

Then, as 36·2 lbs : 108·5 lbs. :: 1000 oz. : 2997 ounces, the specific gravity of the stone = *Answer.*

CASE II.

When the Body is lighter than Water, so that it will not sink.

Ex. 4. 120 lbs. — 80 lbs. = 40 pounds of granite, loss of weight in the water; and, (20 lbs. + 120 lbs.) — 16⅔ lbs. = 123⅓ pounds, loss of weight of the compound in water.

Then, as 123⅓ lbs. — 40 lbs. : 20 lbs. :: 1000 oz. : 240 ounces, the specific gravity of the cork = *Answer.*

CASE III.

For a Fluid of any Kind.

Ex. 6. 166⅝ oz. ÷ 42¾ oz. = 123⅞ ounces, loss of weight of the lignum vitæ in the fluid.

Then as 166⅝ oz. : 123⅞ oz. :: 1333 oz. : 991 ounces, the specific gravity of the fluid = *Answer.*

PROBLEM XXXI.

The linear Dimensions, or Magnitude of a Body, and its specific Gravity, being given, to find its Weight.

Ex. 2. By Rule 1, Prob. XII. Solids. $3\cdot5^3 \times \cdot5236 = 22\cdot44935$ cubic inches, the solidity of the cast-iron shot:

Then, as 1728 cub. in. : 22·44935 cub. in. :: 7645 oz. : 99·32018 ounces = 6·20751 pounds, the weight of the cast-iron shot = *Answer.*

Ex. 3. Here 9 in. − (1½ in. × 2) = 6 inches, the internal diameter.

Then, by Rule 1. Prob. XII. Solids.

$$9^3 \times \cdot5236 = 381\cdot7044 \left\{ \begin{array}{l} \text{cubic inches, the} \\ \text{solidity of the} \\ \text{whole.} \end{array} \right.$$

Again. $$6^3 \times \cdot5236 = 113\cdot0976 \left\{ \begin{array}{l} \text{cubic inches, the} \\ \text{solidity of the} \\ \text{vacuity.} \end{array} \right.$$

The difference = 268·6068 cubic inches, the solidity of the shell.

Then, as 1728 cub. in. : 268·6068 cub. in. :: 7645 oz. : 1188·3674 ounces = 74·2729 pounds, the weight of the cast-iron shell = *Answer.*

Or, get the solidity of the shell thus:

Here 9 in. − (1½ in. × 2) = 6 inches, the internal diameter.

Then, by Example 1, Prob. XLI. Solids. $(\overline{9 + 6}|^2 - \overline{9 \times 6}) \times \overline{9 - 6} \times \cdot5236 = 268\cdot6068$ cubic inches, the solidity of the shell, as before.

Ex. 4. *(See Fig.* 10 *of this Key.)* In this question for " diameter" read " radius."

*As AB + BD = (AD) : AB :: DE : BC; *viz.* as 76 ft. + 5 ft. = (81 ft.) : 76 ft. :: 3⅓ ft. : 3·127572 feet, the lower chord of the front of the stone.

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* Euclid, 4 p. vi. b.

$$\left.\begin{array}{l}\text{Now, by Rule, Prob. X. Solids.}\\ \underline{(DE + \tfrac{1}{2}\,BC)} \times DF = (3\tfrac{1}{3} + \\ 3{\cdot}127572 \div 2) \times 4 \;.\;.\;.\;.\;.\end{array}\right\} = 19{\cdot}5884772$$

$$\left.\begin{array}{l}(BC \underline{+ \tfrac{1}{2}\,DE}) \times CH = (3{\cdot}1275720 \\ + 3\tfrac{1}{3} \div 2) \times 4 \;.\;.\;.\;.\;.\end{array}\right\} = 19{\cdot}1769544$$

$$\text{The sum} = 38{\cdot}7654316$$

Then, 38·76543 × (5 ft. ÷ 3) = 64·60905 cubic feet, which × by 1728 = 111644·4384 cubic inches, the solidity of the keystone.

Then, as 1728 cub. in. : 111644·4384 cub. in. :: 2570 oz. : 166045·2585 oz. = 10377·8286 lbs. = 4 tons 12 cwt. 2 qrs. 17·8286 pounds = *Answer.*

The whole of these Answers agree with those in the work.

Solution, considering 76 *feet as the diameter.**

†As AB + BD = (AD) : AB :: DE : BC; *viz.* as 38ft. + 5ft. = (43 ft.) : 38 ft. :: 3⅓ ft. : 2·945736 feet, the lower chord of the front of the stone.

$$\left.\begin{array}{l}\text{Now, by Rule, Prob. X. Solids.}\\ \underline{(DE + \tfrac{1}{2}\,BC)} \times DF = (3\tfrac{1}{3} + \\ 2{\cdot}945736 \div 2) \times 4 \;.\;.\;.\;.\end{array}\right\} = 19{\cdot}224804$$

$$\left.\begin{array}{l}(\underline{BC + \tfrac{1}{2}}\,DE) \times CH = (2{\cdot}945736 \\ + 3\tfrac{1}{3} \div 2) \times 4 \;.\;.\;.\;.\;.\end{array}\right\} = 18{\cdot}449608$$

$$\text{The sum} = 37{\cdot}674412$$

Then, 37·674412 × (5 ft. ÷ 3) = 62·790686 cubic feet, which × by 1728 = 108502·305408 cubic inches, the solidity of the keystone.

Then, as 1728 cub. in. : 108502·305408 cub. in. :: 2570 oz. : 161372·06302 oz. = 4 tons 10 cwt. 0 qr. 5lbs. 12·06302 ounces, the weight of the keystone = *Answer.*

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* In his solution of this example, Mr. Keith has made use of 76 feet as the radius instead of the diameter, which it really is. *See* Dr. Hutton's *Mathematical Dictionary*, vol. i. p. 252. Edition 1815.

† Euclid, 4 p. vi. b.

Ex. 5. By Rule 2, Prob. XII. Superficies. $32^2 \times {\cdot}7854 = 804{\cdot}2496$ sq. inches, the area of the base of the cylinder; and, by Rule, Prob. IV. Solids. $804{\cdot}2496 \times 14$ in. $= 11259{\cdot}4944$ cubic inches, the solidity of a cylinder whose diameter is 32 inches, and depth 14 inches, equal to the quantity of water displaced by the glass, which is equal to its solidity = 1*st Answer.*

Then, as 1728 cub. in. : 11259·4944 cub. in. :: 2922 oz. : 19039·4922 ounces, the weight of the flint glass = *2nd Answer*

PROBLEM XXXII.

The specific Gravity of a Body, and its Weight, being given, to find its Solidity.

Ex. 2. As 7645 oz. : 42 lbs. :: 1728 cub. in. : 151·8922171353826030 cubic inches, the solidity of the cast-iron shot = 1*st Answer.*

Then, 151·8922171353826030 ÷ by ·5236 = 290·092087729, the cube of the diameter; hence, $\sqrt[3]{290{\cdot}092087729} = 6{\cdot}6198$, the diameter of the cast-iron shot = *2nd Answer.*

Ex. 3. As 930 oz. : 16 oz. :: 1728 cub. in : 29·7 cubic inches of gunpowder = *Answer.*

PROBLEM XXXIII.

To find the Quantities of two Ingredients in a given Compound.

Ex. 3. (*See* the demonstration to this Problem.)

$$\text{Let } \frac{w}{c}, \frac{w}{g}, \frac{w}{l} = \frac{128}{16640}, \frac{128}{19350}, \frac{128}{10510} = \frac{1}{130}, \frac{64}{9675}, \frac{64}{5255}$$

$$\frac{1}{130}\left|\begin{array}{l} \dfrac{64}{9675} \\[2ex] \dfrac{64}{5255} \end{array}\right] \begin{array}{ll} \dfrac{64}{5255} - \dfrac{1}{130} = \dfrac{613}{136630} & \left\{\text{the greater difference.}\right. \\[2ex] \dfrac{1}{130} - \dfrac{64}{9675} = \dfrac{271}{251550} & \left\{\text{the less difference.}\right. \\[2ex] \hline \qquad\qquad\quad \dfrac{56576}{10168425} & \left\{\text{the sum of the diff.}\right. \end{array}$$

Then, $\dfrac{\overset{\text{Sum of Diff.}}{56576}}{10168425} : \dfrac{\overset{\text{Greater Diff.}}{613}}{136630} :: 128$ lbs. : $\dfrac{10168425 \times 613 \times 128}{56576 \times 136630} = \dfrac{1186155}{11492} = 103{\cdot}21571$ pounds of gold, the heavier ingredient = 1*st Answer*.

Also, $\dfrac{\overset{\text{Sum of Diff.}}{56576}}{10168425} : \dfrac{\overset{\text{Less Diff.}}{271}}{251550} :: 128$ lbs. : $\dfrac{10168425 \times 271 \times 128}{56576 \times 251550} = \dfrac{284821}{11492} = 24{\cdot}78428$ pounds of silver, the lighter ingredient = 2*nd Answer*.

**Or thus, the same as Example* 1:

16640 oz. : 128 lbs. :: 1 cub. ft. : ·007692307 cubic feet, the solidity of the compound.

---

* **This Example is better explained by the following equation:**

Let x denote the number of pounds of gold in the compound.

y denote the number of pounds of silver in the compound.

And a denote the weight of the whole compound in pounds.

∴. $16\,x$ will be the number of ounces of gold.

and $16\,y$ will be the number of ounces of silver.

Then, as $19350 : 16\,x :: 1$ ft. : $\dfrac{16\,x}{19350} = \dfrac{8\,x}{9675}$, the solid content of the gold;

As $10510 : 16\,y :: 1$ ft. : $\dfrac{16\,y}{10510} = \dfrac{8\,y}{5255}$, the solid content of the silver;

And as $16640 : 16\,a :: 1$ ft. : $\dfrac{16\,a}{16640} = \dfrac{a}{1040}$ the solid content of the whole mass or compound.

And we shall obviously have $x + y = a$.

And $\dfrac{8\,x}{9675} + \dfrac{8\,y}{5255} = \dfrac{a}{1040}$.

The latter equation, when cleared of fractions, is

$$1748864\,x + 3219840\,y = 2033685\,a$$

The first equation multiplied by 1748864 is $\left\{ 1748864\,x + 1748864\,y = 1748864\,a \right.$

Difference = * $1470976\,y = 284821\,a = 284821 \times 128$

19350 oz. : 128 lbs. :: 1 cub. ft. : ·006614987 cubic feet, the solidity of the gold.

10510 oz. : 128 lbs. :: 1 cub. ft. : ·012178877 cubic feet, the solidity of the silver.

$$\begin{array}{lrcl} & & & \text{Cubic Feet.} \\ \text{Then,} & \cdot007692307 - \cdot006614987 & = & \cdot001077320 \\ \text{And} & \cdot012178877 - \cdot007692307 & = & \cdot004486570 \\ \hline & \text{The sum of the differences} & = & \cdot005563890 \end{array}$$

Then, ·005563890 cub. ft : ·004486570 cub. ft. :: 128lbs. : 103·2157 pounds of gold, the heavier ingredient = 1*st Answer*, as before.

∴ 128lbs.—103·2157lbs. = 24·7843 pounds of silver, the lighter ingredient = 2 *Answer*, as before.

Or thus, by Rule 2:

(16640 oz. — 10510 oz.) × 19350 oz. = 118615500 ounces, the first product.

And (19350 oz. — 10510 oz.) × 16640 oz. = 147097600 ounces, the second product.

Then, 147097600 oz. : 118615500 oz. :: 128 lbs. : 103·2157 pounds of gold, the heavier ingredient = 1*st Answer*, as before.

And 128 lbs. — 103·2157 lbs. = 24·7843 pounds of silver, the lighter ingredient = 2*nd Answer*, as before.

Ex. 3. [*]From 9 ounces, $\left\{\begin{array}{l}\text{the weight of water dis-}\\ \text{placed by the silver.}\end{array}\right.$

Take 6 ounces, $\left\{\begin{array}{l}\text{the weight of water dis-}\\ \text{placed by the crown.}\end{array}\right.$

3 ounces, the remainder.

---

whence $y = \frac{284821 \times 128}{1470976} = \frac{284821}{11492} = 24{\cdot}78428$ pounds, the quantity of silver in the vase.

And $x = a - y = 128 - 24{\cdot}78428 = 103{\cdot}21571$ pounds, the quantity of gold in the vase.

[*] Let a denote the weight of the crown in ounces;
x denote the ounces of gold in the crown;
and y denote the ounces of silver in the crown.

From 6 ounces, { the weight of water displaced by the crown,
Take 5 ounces, { the weight of water displaced by the gold.
1 ounce, the remainder.

The sum of these remainders is 4 ounces.

Then, as 4 oz. : 3 oz. :: 100 oz. : 75 ounces of gold.

And as 4 oz. : 1 oz. :: 100 oz. : 25 ounces of silver.

∴ the crown consisted of 75 ounces of gold and 25 ounces of silver.

PROBLEM XXXIV.

To find how many Inches a floating Body of given Dimensions will sink in a Fluid.

Ex. 2. As 1000 oz. : 600 oz. :: 1728 cub. in. : 1036·8 cubic inches of elm immersed; consequently 1036·8 ÷ by (12 in. × 12 in.) = 7·2 inches, the depth it will sink = *Answer.*

Ex. 3. By Rule, Prob. II. Solids. 40 × 3 × 3 = 360 cubic feet, the solidity of the yellow fir.

Then, by Rule, Prob. XXXI. Solids. 1 cub. ft. : 360 cub. ft. :: 657 oz. : 236520 ounces, the weight of the yellow fir.

Then, as $a : x :: 5 : \frac{5x}{a}$ = the weight of water displaced by the gold.

And as $a : y :: 9 : \frac{9y}{a}$ = the weight of water displaced by the silver.

∴ we shall have $x + y = a$, and $\frac{5x}{a} + \frac{9y}{a} = 6$.

$$\begin{aligned} \text{Or, } 5x + 9y &= 6a \\ 5x + 5y &= 5a \\ \hline \text{The difference} = \quad 4y &= a \end{aligned}$$

∴ $y = \frac{a}{4} = \frac{100}{4} = 25$ ounces of silver in the crown;

And $x = a - y = 100 - 25 = 75$ ounces of gold in the crown, Then, by the Rule. 1030 oz. : 236520 oz. :: 1728 cub. in. : 396802·485436 cubic inches of yellow fir immersed; consequently, 396802·485436 ÷ by (480 in. × 36 in.) = 22·9631067 inches, the depth it will sink = *Answer.*

PROBLEM XXXV.

To find what Weight may be attached to a floating Body, so that it may be just covered with the Fluid.

Ex. 2. 1 cub. ft. × (1030 oz. — 240 oz.) = 790 oz. = 49⅜ pounds weight, to immerse the cork = *Answer.*

Ex. 3. Here half an anker of brandy = 5 gallons, and a gallon of brandy = 231 cubic inches; hence, 231 × 5 = 1155 cubic inches, the content of the half anker, and 1155 + 216 = 1371 cubic inches, the solid content of the oaken cask and brandy.

Then, by Rule, Prob. XXXI. Solids. 1728 cub. in. : 1371 cub. in :: 1030 oz. : 817·204861′ ounces, the weight of sea water equal in bulk to the cask and brandy.

$$\therefore \text{ as } \overset{\text{Cubic Inches}}{1728} : \overset{\text{Cubic Inches.}}{216} :: \overset{\text{Ounces.}}{932} : \overset{\text{Ounces.}}{116\cdot500000} \left\{\begin{array}{l}\text{the weight}\\ \text{of the oak-}\\ \text{en cask.}\end{array}\right.$$

Again,

$$\text{as } 1728 : 1155 :: 927 : 619\cdot609375 \left\{\begin{array}{l}\text{weight of}\\ \text{brandy.}\end{array}\right.$$

The sum = 736·109375 ounces, the weight of brandy and cask.

Then, the weight of the sea water = 817·204861′ ounces,

And the weight of the brandy and cask . . . . } = 736·109375 ounces,

The difference = 81·095486 ounces of sea water.

Now, the specific gravity of lead = 11325 ounces,

And the specific gravity of sea water = 1030 ounces,

The difference = 10295 ounces of lead.

Then, as 10295 oz. : 81·095486 oz. :: 11325 oz. : 89·208973 ounces avoirdupoise = 5lbs. 9·208973 oz., the weight of lead necessary to immerse the half-anker of brandy in sea water = *Answer.*

PROBLEM XXXVI.

To find the Solidity of a Body, lighter than a Fluid, which will be sufficient to prevent a Body, much heavier than the Fluid, from sinking.

Ex. 2. Here 49⅜ lbs. = 790 ounces, the weight of the body to be floated in sea water.

Then, by Rule, Prob. XXXII. Solids. 11325 oz. : 790 oz. :: 1728 cub. in. : 120·540397 cubic inches, the solidity of the lead.

By Rule, Prob. XXXI. Solids. 1728 cub. in. : 120·540397 cub. in. :: 1030 oz. : 71·849889 ounces, the weight of sea water equal in bulk to that of the lead.

Now, by the Rule. (1030 oz. — 240 oz.) : (790 oz. — 71·849889 oz.) :: 1728 cub. in. : 1570·8397364 cubic inches of cork, requisite to keep the lead afloat = *Answer.*

Ex. 3. By Rule, Prob. XXXII. Solids. As 1333 oz. : 146 oz. :: 1728 cub. in. : 189·263315 cubic inches, the solidity of the lignum vitæ.

Then, by Rule, Prob. XXXI. Solids. 1728 cub. in. : 189·263315 cub. in. :: 1000 oz. : 109·527381 ounces, the weight of common water equal in bulk to that of the lignum vitæ.

Now, by the Rule. (1000 oz. — 569 oz.) : (146 oz. — 109·527381 oz.) :: 1728 cub. in. : 146·228968 cubic inches of white fir, requisite to keep the lignum vitæ afloat = 1st *Answer.*

Now, by Rule, Prob. IV. Timb. Measure. 146·228968 cub. in. ÷ by 9 (the area of the end) = 16·247663 = 16¼ inches long *nearly* = *2nd Answer.*

WEIGHT AND DIMENSIONS OF BALLS AND SHELLS.

PROBLEM XXXVII.

The Diameter of an Iron Shot being given, to find its Weight.[*]

Ex. 2. Here $7^3 = 343$ inches, the cube of the diameter.

Then, as 64 in. : 343 in. :: 9 lbs. : 48·234 pounds, the weight of the iron shot = *Answer.*

PROBLEM XXXVIII.

The Diameter of a leaden Ball being given, to find its Weight.[†]

Ex. 2. Here $5{\cdot}75^3 = 190{\cdot}109375$ inches, the cube of the diameter.

Then, as 289 in. : 190·109375 in. :: 64 lbs. : 42·10037 pounds, the weight of the leaden ball = *Answer.*

---

[*] ***Demonstration.*** **The Weight of an iron shot, whose diameter is 4 inches, is known to be 9 lbs.; and as spheres are to each other as the cubes of their diameters, ∴ as 4^3 : D^3 (diameter being $= D$) :: 9lbs. : $\frac{D^3 \times 9}{64}$, the weight of the iron shot, which is the rule.**

[†] ***Demonstration.*** **The weight of a leaden ball, of 4½ inches in diameter is known to be 17 lbs.; hence, as $\overline{4\frac{1}{4}}|^3$: D^3 (diameter being $= D$) :: 17 lbs. : $\frac{64 \times D^3 \times 17}{4913} = \frac{64 \times D^3}{289}$. the weight of the leaden ball, which is the rule.**

PROBLEM XXXIX.

The Weight of an Iron Ball being given, to find its Diameter.

Ex. 2. As 9 lbs. : 24 lbs. :: 64 in. : $\frac{64 \times 24}{9}$ $= \frac{64}{27} \times 72$, the cube of the diameter.

Then, $\sqrt[3]{\frac{64}{27} \times 72} = \frac{4}{3} \sqrt[3]{72} = \frac{8}{3} \sqrt[3]{9} =$ 5·5468898 inches, the diameter of the iron ball = *Answer.*

PROBLEM XL.

The Weight of a Leaden Ball being given, to find its Diameter.

Ex. 2. As 64 lbs. : 8 lbs. :: 289 in. : $\frac{8 \times 289}{64}$ $= \frac{8}{64} \times 289$, the cube of the diameter.

Then, $\sqrt[3]{\frac{8}{64} \times 289} = \frac{1}{2} \sqrt[3]{289} =$ 3·305744 inches, the diameter of the leaden ball = *Answer.*

PROBLEM XLI.

The external and internal Diameters of an Iron Shell being given, to find its Weight.

Ex. 2. $\overline{9{\cdot}8 + 7}|^{2} - (9{\cdot}8 \times 7) \times (9{\cdot}8 - 7) \times \frac{9}{64}$ = 84·12075 pounds weight = *Answer.*

MENSURATION OF TIMBER AND OF ARTIFICERS' WORK.

2.—OF TIMBER MEASURE.

PROBLEM I.

To find the superficial Content of a Board or Plank.

```
            (2.)                              (3.)
 In.         Feet.  In.             In.        Feet.  In.
| 3 | ¼ |    8  .  6               | 2 | ⅙ |   12  .  6
|   |   |    2  .  1½              |   |   |    2  .  1
            ---------                         ---------
            10  .  7½                         14  .  7 = Answer.
                   2½                         ---------
            ---------
             5  .  3¾
            21  .  3
            ---------
         12)26  .  6¾
            ---------
            2s. 2½d. ¼ = Answer.
            ---------
```

Ex. 4. ⅛ (·5 + 1 + 1 + 1·5 + 2 + 2 + 1 + 2) = 1⅜, the mean breadth, and 1⅜ × 14 = 19¼ square feet, the content of the plank = *Answer.*

(5.)

In. In.

$13\frac{1}{2} \times 2 = 27{\cdot}00$

$14\frac{1}{4} \times 1 = 14{\cdot}25$

$18 \times 2 = 36{\cdot}00$

$11\frac{1}{4} \times 2 = 22{\cdot}50$

7)99·75

12)14·25 inches, the mean breadth.

1·1875 feet, ditto.

17·5 feet, the length.

20·78125 square feet, the content of each [plank.

5

103·90625 sq. feet, the content of the five [planks.

Then, as 1 sq. ft. : 103·90625 sq. ft. :: 3*d.* : £1. 5*s.* 11½*d.* ·875, the value of the five oaken planks = *Answer.*

PROBLEM II.

Given the Breadth of a rectangular Plank in Inches, to find how much in Length will make a Foot, or any other assigned Quantity.

Ex. 1. 144 sq. in. ÷ 19 in. = 7·5789 inches in length = *Answer.*

Ex. 2. 1½ yd. × 1296 (in a sq. yd.) = 1944 sq. inches, the area. Then, 1944 ÷ by 26 in· = 74·7692 inches = 6·2307 feet, the distance from the end to be cut off = *Answer.*

Inches.

PROBLEM III.

To find the solid Content of squared or four-sided Timber.

Ex. 2. By Rule 1. $\frac{1}{2}(1\frac{1}{4} + 1\frac{1}{4}) = 1\frac{1}{4}$ feet, the quarter girt.

Then, $\overline{1\frac{1}{4}}|^2 \times 18$ ft. $= 28\frac{1}{8}$ solid feet, the content of the timber = *Answer.*

By Rule 2. $\overline{1\frac{1}{4}}|^2 \times 18$ ft. $= 28\frac{1}{8}$ solid feet = *Answer*, as before.

Ex. 3. By Rule 1. $\frac{1}{2}(25 + 9) = 17$ inches, the quarter girt.

Then, $17^2 \times 20$ ft. $= 5780$, which ÷ 144 = 40·138′ solid feet, the content of the timber = *Answer*

By Rule 2. (*See* Note.) $\frac{1}{2}(25 + 9) \times \frac{1}{2}(25 + 9) \times 20 = 5780$; which ÷ by 144 = 40·138′ solid feet, the content of the timber = *Answer*, as before.

By Rule 3. $(25 + 9) \times (25 + 9) = 1156$, the product of the sum of the breadths and depths of both ends.

And $(25 \times 25) + (9 \times 9) = 706$, the sum of the products of the breadth and depth of each end.

The sum = 1862.

Then, $(1862 \div 6) \times 20$ ft. $= 6206{\cdot}6'$, which ÷ by 144 = 43·10185 solid feet, the content of the timber = *Answer.*

Ex. 4. By Rule 1. (*See* Note.) $\frac{1}{2}(32 + 10) + \frac{1}{2}(20 + 6) = 34$ inches, the breadth and depth in the middle, and 34 ÷ by 2 = 17 inches, the quarter girt. Then $17^2 \times 18$ ft. $= 5202$, which ÷ by 144 = 36·125 solid feet, the content of the timber = *Answer.*

By Rule 2. (*See* Note.) $\frac{1}{2}(32 + 10) \times \frac{1}{2}(20 + 6) \times 18$ ft. $= 4914$, which ÷ by 144 = 34·125 solid feet, the content of the timber = *Answer.*

By Rule 3. $(32 + 10) \times (20 + 6) = 1092$, the product of the sum of the breadths and depths of both ends.

And $(32 \times 20) + (10 \times 6) = 700$, the sum of the products of the breadth and depth of each end.

The sum = 1792.

Then, (1792 ÷ 6) × 18 ft. = 5376, which ÷ by 144 = 37·3′ solid feet, the content of the timber = *Answer.*

Ex. 5. By Rule 1. (*See* Note.) ½ (30 + 15) + ½ (18 + 7) = 35 inches, the breadth and depth in the middle; and 35 ÷ 2 = 17·5 inches, the quarter girt. Then, $17{\cdot}5^2 \times 30$ ft. = 9187·5, which ÷ by 144 = 63·8 solid feet, the content of the timber = *Answer.*

By Rule 2. (*See* Note.) ½ (30 + 15) × ½(18 + 7) × 30 ft. = 8437·5, which ÷ 144 = 58·59375 solid feet, the content of the timber = *Answer.*

By Rule 3. (30 + 15) × (18 + 7) = 1125, the product of the sum of the breadths and depths of both ends.

And (30 × 18) + (15 × 7) = 645, the sum of the products of the breadth and depth of each end.

The sum = 1770.

Then, (1770 ÷ 6) × 30 ft. = 8850, which ÷ by 144 = 61·4583 solid feet, the content of the timber = *Answer.*

PROBLEM IV.

To find how much in Length will make a solid Foot, or any other assigned Quantity, of squared Timber, of equal Dimensions from end to end.

Ex. 1. Here 18^2 = 324 sq. inches, the area of the end. Then, 1728 ÷ 324 = 5⅓ inches, the length that will make a solid foot = *Answer.*

Ex. 2. Here 22 × 15 = 330 sq. inches, the area of the end. Then, 1728 ÷ 330 = 5·23 inches, the length that will make a solid foot = *Answer.*

PROBLEM V.

To find the Solidity of round or unsquared Timber.

Ex. 2. By Rule 1, and Note. $\frac{21{\cdot}5^2 \times 20 \text{ ft.}}{144} =$ 64·2 solid feet, the content of the timber = *Answer.*

Ex. 3. By Rule 1, and Note. $\frac{17{\cdot}75^2 \times 24 \text{ ft.}}{144} =$ 52·51 solid feet, the content of the timber = *Answer.*

Ex. 4. $\frac{1}{5}(9{\cdot}43 + 7{\cdot}92 + 6{\cdot}15 + 4{\cdot}74 + 3{\cdot}16) =$ 6·28 feet, the mean girt, which ÷ by 4 = 1·57 feet, the quarter girt.

Then, by Rule 1. $1{\cdot}57^2 \times 17\frac{1}{4}$ ft. = 42·5195 solid feet, the content of the tree = *Answer.*

Ex. 5. By Rule 1, and Note.

	Feet	Solid Feet.
No. 1.	$(18{\cdot}5^2 \times 45) \div 144 =$	106·953125
No. 2.	$(20^2 \times 17{\cdot}83') \div 144 =$	49·5370′37′
	$(11{\cdot}25^2 \times 22{\cdot}5) \div 144 =$	19·775390
	$(10{\cdot}5^2 \times 13) \div 144 =$	9·953125
No. 3.	$(32^2 \times 10{\cdot}5) \div 144 =$	74·6′66666
	$(25{\cdot}5^2 \times 17) \div 144 =$	76·765625
	$(20{\cdot}5^2 \times 7) \div 144 =$	20·428819
	$(12{\cdot}5^2 \times 14) \div 144 =$	15·190972′
No. 4.	$(39^2 \times 12{\cdot}5) \div 144 =$	132·031250
	$(32^2 \times 17) \div 144 =$	120·8′88888
	$(19{\cdot}5^2 \times 13) \div 144 =$	34·328125
	$(19{\cdot}5^2 \times 16) \div 144 =$	42·250000
	$(10{\cdot}5^2 \times 17{\cdot}5) \div 144 =$	13·398437
The content of the four *elm* trees =		716·167459

Then, as 1 sol. ft. : 716·167459 sol. ft. :: 2*s*. 7*d*. : £92. 10*s*. 1*d*. ·191229, the value of the four *elm* trees = *Answer*.

Or thus, by Rule 2, and Table.

			Solid Feet
No. 1.	45 × 2·376	=	106·920
No. 2.	17·83′ × 2·7′77	=	49·537
	22·5 × ·878	=	19·755
	13 × ·766	=	9·958
No. 3.	10·5 × (1·7′77 × 4) (*see* the Note.)	=	74·6′66
	17 × 4·516	=	76·772
	7 × 2·918	=	20·426
	14 × 1·085	=	15·190
No 4.	12·5 × (2·640 × 4) (*see* the Note)	=	132·000
	17 × (1·7′77 × 4) (*see* the Note)	=	120·8′88
	13 × 2·640	=	34·320
	16 × 2·640	=	42·240
	17·5 × ·766	=	13·405
	The content of the four *elm* trees	=	716·077

Then, as 1 sol. ft. : 716·077 sol. ft. :: 2*s*. 7*d*. : £92. 9*s*. 10¼*d*.·548, the value of the four *elm* trees = *Answer*.

III.—OF CARPENTERS' AND JOINERS' WORK.

1.—*Of Flooring.*

It would be requisite for the learner to commit the following rule to memory, as it is so essential in squaring dimensions.

Rule.

Feet multiplied into feet give feet.
Feet multiplied into inches give inches.
Feet multiplied into seconds give seconds.
Inches multiplied into inches give seconds.
Inches multiplied into seconds give thirds.
Seconds multiplied into seconds give fourths.

Ex 2. 53 ft. 6 in. × 47 ft. 9 in. = 2554 sq. ft. 7 in. 6 sec., which ÷ by 100 = 25 sqrs., 54 sq. feet, 7 inches, 6 seconds, the contents of the floor = *Answer.*

Ex. 3. 36 ft. 3 in. × 16 ft. 6 in. = 598 sq. ft. 1 in. 6 sec., which ÷ 100 = 5 squares, 98 sq. feet, 1 inch, 6 seconds, the content of the floor = *Answer.*

Ex. 4.

Feet. Inches.	Inches. Sec. Thirds.	Inches. Sec.	Inches. Sec
1 . 2	6 . 6 . 0	8 . 6	2 . 6
1 . 0	3 . 0 . 0	4 . 0	4 . 0
1 . 2	1 . 7 . 6	2 . 10	10
20	20	9	6
23 . 4	2 . 8 . 6 . 0	2 . 1 . 6	5 . 0
	8	8	24
	21 . 8 . 0 . 0	17 . 0 . 0	10 . 0 . 0

Then, 23 ft. 4 in. + 21 ft. 8 in. + 17 ft. + 10 ft. = 72 solid feet, the content of the whole = *Answer.*

(Ex 5)

Feet. Inches.	Feet. Inches.	Feet. Inches.	Feet. Inches. Sec.	Feet. Inches.	Feet. Inches. Sec.	Feet. In. Sec.
20 . 8	6 . 0	6 . 0	5 . 8 . 0	5 . 2	10 . 6 . 0	54 . 0 . 0
16 . 9	4 . 6	5 . 4	4 . 8 . 0	4 . 0	8 . 9 . 0	64 . 0 . 0
						52 . 10 . 8
330 . 8	24 . 0	30 . 0	22 . 8 . 0	20 . 8	84 . 0 . 0	20 . 8 . 0
15 . 6	3 . 0	2 . 0	3 . 9 . 4		7 . 10 . 6	367 . 6 . 0
346 . 2	27 . 0	32 . 0	26 . 5 . 4		91 . 10 . 6	559 . 0 . 8
4	2	2	2		4	
1384 . 8	54 . 0	64 . 0	52 . 10 . 8		367 . 6 . 0	

Square Feet. Inches. Sec.
Then, 1384 . 8 . 0 the content of the whole;
and 559 . 0 . 8 the content of the 7 fire-places and the well-hole for the stairs.

The difference = 825 . 7 . 4, the superficial content of the four floors.

Then, as 100 sq. ft. : 825 sq. ft. 7 in. 4 sec. :: £6. 10*s*. : £53. 13*s*. 3½*d*. ·13′, the expense of flooring the four floors = *Answer*.

2.—*Of Partitioning.*

Ex. 1. 82 ft. 6 in. × 12 ft. 3 in. = 1010 sq. ft. 7 in. 6 sec., which ÷ 100 = 10 sqrs. 10 sq. ft. 7 in. 6 sec., the content of the partition = *Answer.*

Ex. 2. 91 ft. 9 in. × 11 ft. 3 in. = 1032 sq. ft. 2 in. 3 sec., which ÷ by 100 = 10 sqrs. 32 sq. ft. 2 in. 3 sec., the content of the partition = *Answer.*

3.—*Of Roofing.*

Feet. Inches.

Ex. 2. 52 . 8, the length within the walls.

30 . 6, the breadth within the walls.

1580 . 0

26 . 4

2)1606 . 4, the flat.

803 . 2, half the flat.

2409 . 6, which ÷ by 100 = 24 sqrs. 9 sq. feet, 6 inches, the area of the roof.

Then, as 100 sq. ft. : 2409 sq. ft. 6 in. :: 10½*s.* : £12. 12*s.* 11¾*d.* ·88, the expense of roofing the house = *Answer.*

Feet. Inches.

Ex. 3. 2)20 . 6, the breadth within the walls.

10 . 3, the half-breadth within the walls.

30 . 9, the breadth of the roof.

40 . 6, the length of the roof.

1230 . 0

15 . 4 . 6

1245 . 4 . 6; which ÷ by 100 = 12 sqrs. 45 sq. feet, 4 inches, 6 sec.; or = 12·45375 squares, the content of the roof = *Answer*,

(Ex. 4.)

Feet. In. Feet. In. In.		Feet. In. Sec.		
36 . 0 × 1 . 2 × 9	=	31 . 6 . 0	solidity of the tie-beam D.	*Answer.*
11 . 6 × 1 . 0 × 5	=	4 . 9 . 6	ditto of the king-post A.	
7 . 6 × 0 . 5 × 5 × 2	=	2 . 7 . 3	ditto of the braces BB.	
19 . 0 × 0 . 5 × 10 × 2	=	13 . 2 . 4	ditto of the rafters EE.	
3 . 6 × 0 . 4 × 5 × 2	=	11 . 8	ditto of the struts CC.	
The sum	=	53 . 0 . 9	solidity for workmansh.	
7 . 0 × 0 . 3 × 5 × 2	=	1 . 5 . 6	ditto cut from king-post.	
The difference	=	51 . 7 . 3	ditto for materials.	

4.—*Of Wainscotting.*

Ex. 2. 137 ft. 6 in. × 16 ft. 3 in. = 2234 sq. ft. 4 in. 6 sec., which ÷ by 9 = 248 sq. yards, 2 sq. feet, 4 inches, 6 seconds, the content of the wainscot = *Answer.*

(Ex. 3.)

Feet
69·75, the breadth.
6·25, the height.

2)435·9375 sq. feet, the area of the window-shutters.
217·9687 sq. feet, the half area of ditto.

9)653·9062 sq. feet, the sum at work and half.

72·6562 sq. yards, the content of the window-shutters = *Answer.*

(Ex. 4.)

Feet. In. Feet. In. Feet. In.
7 . 8 × 3 . 6 × 3 = 80 . 6 the area of the three window shutters.
3 . 6 × 7 . 0 = 24 . 6, the area of the door.

The sum = 105 . 0 the area of the door and window shutters.
52 . 6, the half area of ditto ditto.

The sum = 157 . 6, the content of the door and window shutters at work and half.

Feet. Inches. Feet. Inches. Feet. Inches.
83 . 8 × 12 . 6 = 1045 . 10, area of the whole room.
105 . 0 the area of the door and window shutters.

The difference = 940 . 10 the content of the wainscotting.
157 . 6 content of door & window shutters at work & half.

9)1098 . 4, the sum.

122 sq. yds. 4 in. the content of the [whole.

Then, as 1 sq. yd. : 122 sq. yds. 0 sq. ft. 4 in. :: 6*s.* : £36 12*s.* 2½*d.* ⅔, the expense of the whole = *Answer.*

(Ex. 5.)

$\overset{\text{Feet.}}{129}\,.\,\overset{\text{In.}}{6}\times\overset{\text{Feet.}}{16}\,.\,\overset{\text{In.}}{3}$	=	$\overset{\text{Feet.}}{2104}\,.\,\overset{\text{In.}}{4}\,.\,\overset{\text{Sec.}}{6}$	the area of whole room.
$\frac{1}{2}(7\,.\,0\times 3\,.\,9)$	=	13 . 1 . 6	the half work to door.
$\frac{1}{2}(7\,.\,3\times 4\,.\,6\times2)$	=	32 . 7 . 6	the half work to window-shutters.
$(7\,.\,3+1\,.\,3)\times\overset{\text{Feet.}}{1}\,.\,\overset{\text{In.}}{2}\times4$	=	39 . 8 . 0	area of the side cheek-boards.
$4\,.\,6\times1\,.\,2\times4$	=	21 . 0 . 0	area of the end cheek-boards.
$(\overline{7\,.\,0\times2}+\overset{\text{Feet.}}{3}\,.\,\overset{\text{In.}}{9})\times\overset{\text{Feet.}}{1}\,.\,\overset{\text{In.}}{4}$	=	23 . 8 . 0	area of lining boards round the door.
The sum	=	2234 . 5 . 6,	
3 ft. 9 in. × 3 ft.	=	11 . 3 . 0	area of the chimney to be deducted.
		9)2223 . 2 . 6	

247 sq. yds. 2 in. 6 seconds, the content of the whole work.

Then, as 1 sq. yd. : 247 sq. yds. 0 sq. ft. 2 in. 6 sec. :: 3½*s.* : £43. 4*s.* 6¾*d.* $\frac{8}{9}$, the estimate of the charge = *Answer.*

IV.—OF BRICKLAYERS' WORK.

1.—*Of Tiling or Slating.*

Ex. 2. 35 ft. 9 in. × 43 ft. 6 in. = 1555 sq. ft. 1 in. 6 sec., which ÷ by 100 = 15 squares, 55 sq. feet, 1 inch, 6 seconds, the content of the tiling = *Answer.*

Ex. 3.

Feet. Inches. Sec.	
43 . 10 . 0,	the length of the eave.
1 . 4 . 0,	the breadth of the eave.
58 . 5 . 4,	the area of one eave.
2	
116 . 10 . 8,	the area of the two eaves.

Feet. Inches. Sec.
43 . 10 . 0, the length of the barn.
27 . 5 . 0, the breadth of the barn.

2)1201 . 9 . 2, the flat.
600 . 10 . 7, the half flat.
116 . 10 . 8, the area of the two eaves.

1919 . 6 . 5, the content of the tiling.

Then, as 100 sq. ft. : 1919 sq. ft. 6 in. 5 sec. :: 25½*s.* : £24. 9s. 5¾*d.* ·105, the expense of tiling the barn = *Answer.*

2.—*Of Walling.*

Ex 2.
Feet.
245·75, length.
16·5, height.

4054·875
5 half-bricks.

3)20274·375

272)6758·125(24 rods.
544

1318
1088

68)230(3 quarters.
204

26 feet.

Answer—24 rods, 3 quarters, 26 feet.

Ex. 3. ⅓(63 ft. 6 in. × 14 ft. 11 in. × 5) = 1578 sq. ft. 8 in. 2 sec.; which ÷ by 272 = 5 rods, 218 sq. feet, 8 inches, 2 sections, the content of the wall = *Answer*.

Ex. 4. By Rule, Prob. IV. Superficies. ½(15ft. × 45 ft). = 337·5 sq. feet, the area of the gable-end; and ⅓(337·5 × 5) = 562·5 sq. feet, the content of the brick-work at standard thickness; which ÷ by 272 = 2 sq. rods, 18½ sq. feet, the content of the gable-end = *Answer.*

Ex. 5.

Feet. Inches.	Feet. Inches.	Feet Inches. Sec.	Courses
28 · 10	28 : 10	28 · 10 · 0	4)42 · 0
20 · 0	20 · 0	15 · 8 · 0	
			10 · 6
576 · 8	576 · 8	451 · 8 · 8	28 · 10
5	4	3	
			302 · 9
2883 · 4	2306 · 8	1355 · 2 · 0	

2883 · 4, content of the first 20 feet
2306 · 8, ditto of the second 20 feet
1355 · 2, ditto of 15 feet, 8 inches
302 · 9, ditto of the gable
} in half bricks.

3)6847 · 11, the sum.

2282 · 7 · 8″, the content of the wall reduced to [standard thickness

Then, as 272 sq. ft. : 2282 sq. ft. 7 in. 8 sec. :: £5. 16*s.* : £48. 13*s.* 5½*d.* ·960784, the expense of the work = *Answer.*

3.—*Of Chimnies.*

(Ex. 2.)

Feet. Inches.
3 · 6 wide.
4 · 0 high.
14 · 0
6 half bricks.
3)84 · 0
28 · 0 at standard thickness.
3 the three fire-places.
84 · 0 the content of the vacuity of the three fire places at standard thickness.

Feet· In.
14 · 6 the height of the parlour.
12 · 0 the height of the first floor.
10 · 6 the height of the second floor.

37 · 0 the sum of the heights.
7 · 3 the breadth of the breast of the
[chimney.
268 · 3
6 half bricks, depth of the jambs.

3)1609 · 6

536 · 6 at standard thickness.
10 ft. × 6 ft. = 60 · 0 the content of the shaft.

The sum = 596 · 6 the content of the whole at
[standard thickness.
84 · 0 the content of the vacuity of
[three fire-places, at ditto.
The difference = 512 · 6 the measure.
2

1025 · 0 double measure.

272)1025(3 rods.
816

68)209(3 quarters.
204

5 feet.

Answer, 3 sq. rods, 3 quarters of a rod, 5 sq. feet.

V.—OF PLASTERERS' WORK.

Ex. 2. 141·5 ft. × 11¼ ft. = 1591·875 sq. feet; which ÷ by 9 = 176·875 sq. yards, the content of the partitions = *Answer.*

(Ex. 3.)

Feet. Inches.
22 · 7 = 22·583′, the length. } Multiplied.
13 · 11 = 13·916′, the breadth. }

9)314·28472′

34·92052469, &c. sq. yards.
11½ pence.

401·586033 pence = £1. 13*s*. 5½*d*. ·344, the expense of plastering the ceiling = *Answer*.

(Ex. 4.)

Feet. In. Feet. In. Sq. Feet. In. Sec.
234 · 8 × 14 · 6 = 3402 · 8 · 0 area of the partition.
680 · 6 · 4 one-fifth part of the area.

9)4083 · 2 · 4 the sum.

453 sq. yds. 6 sq. feet, 2 inches, 4 seconds, the content for the whitening.

Then, as 1 sq. yd. : 453 sq. yds. 6 sq. ft. 2 in. 4 sec. :: 2*d*. : £3. 15*s*. 7¼*d*. ·5061, the expense for whitening.

Sq. Feet. Inches. Sec.
Again, as above. 3402 · 8 · 0 area of the partition.
680 · 6 · 4 one-fifth part of the area.

9)2722 · 1 · 8 the difference.

302 · 4 · 1 · 8″ the content for the plastering.

Then, as 1 sq. yd. : 302 sq. yds. 4 sq. feet, 1 in. 8 sec. :: 8*d*. : £10. 1*s*. 7½*d*. ·7160, the expense for plastering.

£. s. d.
Hence, 3 · 15 · 7¼ ·5061 the expense for whitening.
10 · 1 · 7½ .7160 the expense for plastering.

13 · 17 · 3 ·2221 expense of the whole = *Answer*.

(Ex. 5.)

Feet. Inches. Feet. Inches. Sq. Feet. In. Sec.

14 · 5 × 9 · 3 × 2 = 266 · 8 · 6 $\Big\{$ the content of the two sides.

13 · 2 × 9 · 3 × 2 = 243 · 7 · 0 $\Big\{$ the content of the two ends.

The sum = 510 · 3 · 6 the area of the walls.

Deduct 7 ft. × 4 ft. = 28 · 0 · 0 content of the door.

9)482 · 3 · 6

53 · 5 · 3 · 6″ the content of the rendering.

Feet. In.

Then, 14 ft. 5 in. the length − 10 in. the cornice . $\Big\}$ = 13 · 7 $\Big\}$ Multiplied.

13 ft. 2 in. the breadth − 10 in. the cornice . $\Big\}$ = 12 · 4

9)167 · 6 · 4″

18 · 5 · 6 · 4″ the content of the ceilings.

Answer, $\Bigg\{$ 53 sq. yds. 5 sq. ft. 3 in. 6 sec. of rendering. and 18 sq. yds. 5 sq. ft. 6 in. 4 sec. of ceiling.

Ex. 6. $\frac{1}{9}$ (275 ft. 5 in. × 105 ft. 6 in.) = 3228 sq. yds., 4 sq. ft. 5 in. 6 sec. the content of the roof.

Then, as 1 sq. yd. : 3228 sq. yds. 4 sq. ft. 5 in. 6 sec. :: 1*s.* : £161. 8*s.* 5$\frac{3}{4}$*d.* $\frac{7}{9}$, the expense = *Answer.*

Feet. In. Feet. In. Feet. Feet. Square Feet.
Ex. 2. 74 · 10 × 11 · 7 = 74·83′ × 11·583′ = 866·8194′, the area of the whole room.
7 · 6 × 3 · 9 = 7·5 × 3·75 = 28·1250′, the area of the door.
6 · 8 × 3 · 4 × 5 = 6·6′ × 3·3′ × 5 = 111·1111′, the area of the shutters.
(8 + 8 + 3·3′ + 3·3′) × 1·16′ × 5 = 132·2222′, the area of the breaks.
1138·2777′, the sum of the areas.
Deduct 6 ft. 9 in. × 5 ft. = 33·7500′, the area of the chimney.
The area of the whole work = 1104·5277′ square feet.
122·7253 sq. yds; which × by 8½*d.*
= 1043·165 pence = £4. 6*s.* 11*d.*·165, the expense of the whole = *Answer.*

Feet. In. Feet. In. Sq. Feet. In. Sec.
Ex. 3. 24 · 6 × 12 · 9 × 2 = 624 · 9 · 0, the area of the two sides of the room.
16 · 3 × 12 · 9 × 2 = 414 · 4 · 6, the area of the two ends of ditto.
7 · 0 × 3 · 6 = 24 · 6 · 0, the area of the door.
7 · 9 × 3 · 6 × 2 = 54 · 3 · 0, the area of the two shutters.
8 · 6 × 1 · 3 × 4 = 42 · 6 · 0, the area of the four sides or the breaks.
3 · 6 × 1 · 3 × 4 = 17 · 6 · 0, the area of the four ends or the breaks.
1177 · 10 · 6, the sum.
Deduct 5 ft. 6 in. × 5 ft. = 27 · 6 · 0, the area of the fire-place.
9)1150 · 4 · 6
127 sq. yds. 7 sq. ft. 4 in. 6 sec. content of the whole work.
Then, as 1 sq. yd. : 127 sq. yds. 7 sq. ft. 4 in. 6 sec. : : 2*d.* × 3 : £3. 3*s.* 10¾*d.*·6′, the expense
= *Answer.*

VII.—OF GLAZIERS' WORK.

Ex. 2. 4 ft. 7 in. 9 sec. × 1 ft. 5 in. 3 sec. × 8 = 53 sq. ft. 5 in. 1 sec. 6 thirds, the content of the eight panes = *Answer*.

Ex. 3. 4 ft. 5 in. 6 sec. × 1 ft. 4 in. 9 sec. × 16 = 99 sq. ft. 6 inches, 10 seconds, the content of the sixteen panes = *Answer*.

Ex. 4. 7 ft. 3 in. × 3 ft. 5 in. = 24 sq. ft. 9 in. 3 seconds, the content of the window = *Answer*.

Ex. 5. (6 ft. 11 in. + 5 ft. 4 in. + 4 ft. 3 in.) × 7 = 115 feet, 6 inches, the length of the whole of the windows; and 115 ft. 6 in. × 3 ft. 6 in. = 404 sq. ft. 3 inches, the area of the whole.

Then, as 1 sq. ft. : 404 sq. ft. 3 in. :: 14½*d*. : £24. 8*s*. 5½*d*.·5, the expense of the glazing = *Answer*.

Ex. 6. By Rule, Prob. IV. Superficies. ½ (16 ft. 9 in. × 12 ft. 6 in.) = 104 sq. feet, 8 inches, 3 seconds, the area of the triangular skylight.

Then, as 1 sq. ft. : 104 sq. ft. 8 in. 3 sec. :: 10*d*. : £4. 7*s*. 2¾*d*.·5, the expense of the triangular skylight = *Answer*.

Ex. 7. 14 ft. 6 in. × 4 ft. 9 in. = 68 sq. feet, 10 inches, 6 seconds, the area of the elliptical fanlight = *Answer*.

VIII.—OF PLUMBERS' WORK.

Ex. 2. 18 ft. 6 in. × 12 ft. 3 in. = 226 sq. ft. 7 inches, 6 seconds, the area of the flat.

Then, as 1 sq. ft. : 226 sq. ft. 7 in. 6 seconds :: 6 lbs. : 12 cwt. 0 qrs. 15¾ lbs., the weight of lead = *Answer*.

Ex. 3. As 1 yd. : 120 yds. :: 18 lbs. : 2160 lbs., the weight of the leaden pipe.

Then, as 1 lb. : 2160 lbs. :: 3*d.* : £27, the expense of the leaden pipe = *Answer.*

Ex. 4. 43 ft. × 32 ft. = 1376 sq. feet, the area of the roof.
57 ft. × 2 ft. = 114 sq. feet, the area of the gutter.
The sum = 1490 sq. feet, the area of the roof and gutter.

Then, as 1 sq. ft. : 1490 sq. ft. :: 8¾ lbs. : 13037½ lbs., the weight of the lead; hence, as 112 lbs. : 13037½ lbs. :: 18*s.* : £104. 15*s.* 3¾*d.*, the expense of covering the gutter and roof = *Answer.*

IX.—OF MASONS' WORK.

Ex. 2. 107 ft. 9 in. × 20 ft. 6 in. = 2208 sq. feet, 10 inches, 6 seconds, the superficial content of the wall = *Answer.*

Ex. 3. $\frac{1}{63}$(112 ft. 3 in. × 16 ft. 6 in.) = 29 sq. rods, 25 sq. feet, 1 inch, 6 seconds, the superficial content of the wall = *Answer.*

Ex. 4. 5 ft. 7 in. × 1 ft. 10 in. = 10 sq. ft. 2 in. 10 seconds, the superficial content of the marble slab.

Then, as 1 sq. ft. : 10 sq. ft. 2 in. 10 sec. :: 6*s.* : £3. 1*s.* 5*d.* the value of the marble slab = *Answer.*

X.—OF PAVERS' WORK.

Ex. 2. $\frac{1}{9}$ (24 ft. 5 in. × 12 ft. 7 in.) = 34 sq. yds. 1 sq. foot, 2 in., 11 seconds, the area of the court-yard.

Then, as 1 sq. yd. : 34 sq. yds. 1 sq. ft. 2 in. 11 sec. :: 3*s.* 4*d.* : £5. 13*s.* 9½*d.* ·09876, the expense of paving the court-yard = *Answer.*

Ex. 3. 62 ft. 7 in. × 44 ft. 5 in. = 2779 sq. ft. 8 in. 11 seconds, the area of the whole yard; and 62 ft. 7 in. × 5 ft. 6 in. = 344 sq. ft. 2 in. 6 seconds, the area of the foot-path.

Then, as 9 sq. ft. : 2779 sq. ft. 8 in. 11 sec. :: 2½*s.* : £38. 12*s.* 1¾*d.* ·24074, the expense of paving the court-yard, at 2*s.* 6*d.* per yard.

Again. As 9 sq. ft. : 344 sq. ft. 2 in. 6 sec. :: 6*d.* : 19*s.* 1¼*d.* ·8′8888, the expense of paving the foot-path, at 6*d.* per yard.

	£.	s.	d.	
Then,	38	12	1¾ ·24074	the expense of paving the court-yard, at 2*s.* 6*d.* per yard.
And		19	1¼·8′8888	the expense of paving the foot-path, at 6*d.* per yard.
Sum =	39	11	3¼ ·12962	the whole cost = *Answer*.

XI.—OF VAULTED AND ARCHED ROOFS, &c.

PROBLEM I.

To find the Solidity of a Circular, Elliptical, or Gothic Vaulted Roof.

Ex. 3. By the Note. 30 ft. × 15 ft. × ·7854 = 353·43 sq. feet, the area of the end. Then, by the Rule. 353·43 × 90 ft. = 31808·7 solid feet, the content of the vacuity = *Answer.*

Ex. 4. By Rule 2, Prob. XIII. Superficies. 76^2 × ·7854 = 4536·4704 sq. feet, the area of the whole circle.

		Square Feet.	
Hence, 4536·4704 ÷ by 2	=	2268·2352	area of the semicircle.
By Rule, Prob. II. Superficies. 76 ft. × 2 ft.	=	152·0000	area of the rectangle.

The sum = 2420·2352, the area of the front of the arch.

Then, by the Rule. 2420·2352 × 44 ft. = 106490·3488 solid feet, the solidity of the vacuity of the arch = *Answer.*

Ex. 5. By Rule 2, Prob. XIII. Superficies. $18^2 \times \cdot 7854 = 254\cdot 4696$ sq. feet, the area of the whole circle; which ÷ by 2 = 127·2348 sq. feet, the area of the semicircle.

Then, by the Rule. 127·2348 × 50 feet = 6361·74 solid feet, the solidity of the vacuity of the circular vault = 1*st Answer.*

Again. By Rule, Prob. II. Superficies. $(\overline{3 \text{ ft.} \times 2} + 18 \text{ ft.}) \times (9 \text{ ft.} + 2 \text{ ft.}) = 264$ sq. feet, the area of the front of the whole, *viz.* the vacuity and materials; hence, by the Rule. 264 × 50 ft. = 13200 solid feet, the solidity of the whole.

Then, 13200 sol. ft. − 6361·74 sol. ft. = 6838·26 solid feet, the solidity of the materials of the circular vault = 2*nd Answer.*

Ex. 6. *$\sqrt{BO^2 - BC^2} = \sqrt{40^2 - 15^2} = \sqrt{1375} = 37\cdot 08099243547$ feet, the height of the vacuity of the Gothic roof CO.

Then, by Rule, Prob. IV. Superficies. $\frac{1}{2}(CO \times AB) = \frac{1}{2}(37\cdot 08099243547 \times 30 \text{ ft.}) = 556\cdot 21488653205$ sq. feet, the area of the triangle ABO.

Then, by Rule 4, and Note to Prob. XVI. Superficies. $(BO \times DE \times \frac{2}{3}) + (DE^3 \div \overline{BO \times 2}) = (40 \times 6 \times \frac{2}{3}) + (6^3 \div \overline{40 \times 2}) = 162\cdot 7$ sq. feet, the area of the segment OBEO; hence, ABO + 2 OBEO $= 556\cdot 21488653205 + (162\cdot 7 \times 2) = 881\cdot 61488653205$ sq. feet, the area of the end of the Gothic roof ABEOA.

Then, by the Rule. ABEOA × 60 ft. = 881·61488653205 × 60 ft. = 52896·893191923 solid feet, the solidity of the vacuity of the roof = 1*st Answer.*

Then, by Rule, Prob. II. Superficies. LK × CG

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* Euclid, 47 p. i. b.

$= 64$ ft. $\times 41{\cdot}08099243547$ ft. $= 2629{\cdot}18351587008$ sq. feet, the area of the section FHKL. Then, by the Rule. FHKL $\times 60$ ft. $= 2629{\cdot}18351587008 \times 60$ ft. $= 157751{\cdot}0109522048$ solid feet, the solidity of the whole, *viz.* the vacuity and materials; hence, $157751{\cdot}0109522048$ sol. ft. $- 52896{\cdot}893191923$ sol. ft. $= 104854{\cdot}1177602818$ solid feet, the solidity of the materials $=$ *2nd Answer.*

PROBLEM II.

To find the Concave or Convex Surface of Circular, Elliptical, or Gothic Vaulted Roofs.

Ex. 1. *$\sqrt{25{\cdot}4^2 + 17{\cdot}0646^2} = \sqrt{936{\cdot}36057316} = 30{\cdot}6$ feet, the chord of half the arch. Then, by Rule 2, Prob. XII. Superficies. $\frac{1}{3}(\overline{30{\cdot}6 \times 8 - 50{\cdot}8}) = 64\frac{2}{3}$ feet, the length of the arch. Then, by the Rule. $64\frac{2}{3}$ ft. $\times 50$ ft. $= 3233\frac{1}{3}$ sq. feet, the concave surface of the vault $=$ *Answer.*

Ex. 2. *$\sqrt{70^2 + 35^2} = \sqrt{6125} = 78{\cdot}262379$ feet, the chord of half the arch. Then, by Rule 2, Prob. XII. Superficies. $\frac{1}{3}(\overline{78{\cdot}262379 \times 8 - 140}) = 162{\cdot}03301$ feet, the length of the arch. Then, by the Rule. $162{\cdot}03301$ ft. $\times 11$ ft. $= 1782{\cdot}36311$ sq. feet, the concave surface of the bridge $=$ *Answer.*

PROBLEM III.

To find the solid Content of a Dome, having the Height and Dimensions of its Base given.

Ex. 2. By Rule 2, and Table, Prob. X. Superficies. $20^2 \times 4{\cdot}828427$ *(tab. area.)* $= 1931{\cdot}3708$ sq. feet, the area of the base. Then by the Rule. $1931{\cdot}3708$

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* Euclid, 47 p. i. b.

× 26 ft. × $\frac{2}{3}$ = 33477·09386 solid feet, the solidity of the octagonal dome = *Answer.*

Ex. 3. By Rule, Prob. XXI. Superficies. 40 ft. × 30 ft. × ·7854 = 942·48 sq. feet, the area of the base of the vacuity. Then, by the Rule. 942·48 × 17·32 ft. × $\frac{2}{3}$ = 10882·5024 solid feet, the solidity of the interior vacuity of the elliptical dome.

Again. $\overline{4\text{ ft.} \times 2}$ + 40 ft. = 48 feet, the outer transverse diameter; and $\overline{4\text{ ft.} \times 2}$ + 30 ft. = 38 feet, the outer conjugate diameter.

Hence, by Rule, Prob. XXI. Superficies. 48 × 38 × ·7854 = 1432·5696 sq. feet, the area of the whole base, *viz.* the vacuity and stone-work.

Then, by the Rule. 1432·5696 × (17·32 ft. + 4 ft.) × $\frac{2}{3}$ = 20361·589248 solid feet, the solidity of the whole, *viz.* the vacuity and stone-work.

Then, 20361·589248 sol. ft. − 10882·5024 sol. ft. = 9479·086848 cubic feet, the solidity of the stone-work = *Answer.*

PROBLEM IV.

To find the Superficies of a Dome, having the Height and Dimensions of its Base given.

Ex. 2. 40 ft. × 30 ft. × 1·5708 = 1884·96 square feet, the concave surface = *Answer.*

PROBLEM V.

To find the solid Content of a Saloon.

Ex. 2. *(See Fig.* 17 *of this Key.)* By Rule 4, *Note* to Prob. XVI. Superficies. (AC × Dm × $\frac{2}{3}$) + (Dm^3 ÷ 2 AC) = (4·5 × ·75 × $\frac{2}{3}$) + (·75^3 ÷ $\overline{4 \cdot 5 \times 2}$) = 2·296875 sq. feet, the area of the segment

ACmA. *$\sqrt{AC^2 - AB^2} = \sqrt{4{\cdot}5^2 - 3{\cdot}2^2} = \sqrt{10{\cdot}01}$ = 3·163858ft. = BC. Then, BC × ½AB = 3·163858 × $\overline{3{\cdot}2 \div 2}$ = 5·0621728 sq. feet, the area of the △ ACB; hence, ACB − ACmA = 5·0621728 − 2·296875 = 2·7652978 sq. feet, the area of the section ABCmA. Then, by the Rule. ABCmA × OePrO = 2·7652978 × 50 ft. = 138·26489 cubic feet, the content of the solid part of the saloon OrPeO = *Answer.*

Ex. 3. (*See Fig.* 17 *of this Key.*) By Rule 2, Prob. XIII. Superficies. $GTF^2 \times {\cdot}7854 = 40^2 \times {\cdot}7854$ = 1256·64 sq. feet, the area of the floor of the room GVFWG, and = the area of the circular part of the room K*y*A*x*K. Then, by Rule, Prob. IV. Solids. GVFWG × FA = 1256·64 × 20 ft. = 25132·8 cub. ft. = FAKGF, the solidity of the room to the spring of the arch of the saloon.

Now, $Em^2 \times {\cdot}7854 = 5^2 \times {\cdot}7854$ = 19·635 sq. feet, the area of the quadrant EA*m*CE; and AB × BC = 5 ft. × 5 ft. = 25 sq. feet, the area of the square ABCEA. Hence, ABCEA — EA*m*CE = 25 − 19·635 = 5·365 sq. feet, the area of the section ABC*m*A. *$\sqrt{EA^2 + EC^2} = \sqrt{5^2 + 5^2} = \sqrt{50}$ = 7·07106 = AC = EB; and ½ (EB − E*m*) = ½ (7·07106 − 5) = 1·03553 = OB. Then, †EB : CB :: OB : HB, *viz.* as 7·07106 ft. : 5 ft. :: 1·03553 ft. : ·732231 ft. Hence, GF − 2 HB = 40 − $\overline{{\cdot}732231 \times 2}$ = 38·535538 ft. = O*n*P, the diameter of the solid part of the saloon taken in the middle. Then, by Rule 1, Prob. XI. Superficies. O*n*P × 3·1416 = 38·535538 × 3·1416 = 121·0632461808 feet = O*e*P*r*O, the circumference of the solid part of the saloon taken in the middle.

Then, by the Rule. O*e*P*r*O × ABC*m*A = 121·0632461808 × 5·365 = 649·504315759992 cub. ft. = O*e*P*r*O, the solid work of the saloon.

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* Euclid, 47 p. i. b. † Euclid, 4 p. vi. b.

By Rule, Prob. IV. Solids. $KyAxK$ × AB = 1256·64 sq. ft. × 5 ft. = 6283·2 cub. ft. = ABLKA, the solidity of the cylindrical part of the room.

Solid Feet.

Hence, the solidity of the cylindrical part ABLKA = 6283·200000000000

And the solidity of the saloon taken in the middle $OePrO$ . . = 649·504315759992

The difference = 5633·695684240008 cubic feet = $AmCMbKA$, the solidity of the vacuity of the saloon.

Solid Feet.

∴ The solidity of the cylindrical part FAKGF = 25132·800000000000

And the solidity of the vacuity of the saloon $AmCMbKA$ . . = 5633·695684240008

The sum = 30766·495684240008 cubic feet, the whole solidity of the room $GFAmCMbKG$ = *Answer.*

PROBLEM VI.

To find the superficial Content of a Saloon.

Ex. 2. *(See Fig.* 17 *of this Key.)* By Rule 1, Prob. XI. Superficies. GTF × 3·1416 = 40 ft. × 3·1416 = 125·664 feet, the circumference of the floor GVFWG; and by Note to Prob. IV. Solids. GVFWG × FA = 125·664 × 20 ft. = 2513·28 sq. ft. = FAKGF, the convex surface of the room to the spring of the arch of the saloon.

Now, by Rule 1, Prob. XI. Superficies. 2 AE × 3·1416 = 5ft. × 2 × 3·1416 = 31·416 feet, the circumference of the circle $ACzhA$; ∴ ¼ $ACzhA$ = 31·416 ÷ 4 = 7·854 feet, the length of the quadrantal arch, or breadth of the curved part of the saloon AmC.

See Example 3 of the preceding Proposition. EB =

7·071 ft. and $Em = 5$ ft. $= CB$; $\therefore mB = 2·071$ ft. *$EB : CB :: mB : vB$; *viz.* as 7·071 ft. : 5 ft. :: 2·071 ft. : 1·4644 ft. Then, $GTF - 2\,vB = 40$ ft. $- \overline{1·4644 \times 2} = 37·0712$ ft. $= mrb$, the diameter of the room at the middle of the quadrantal arch AmC; and, by Rule 1, Prob. XI. Superficies. $mrb \times 3·1416 = 37·0712$ ft. $\times\ 3·1416 = 116·46288192$ feet $= msbnm$, the circumference of the room at the middle of the arch AmC.

Then, by the Rule. $msbnm \times AmC = 116·46288192$ ft. $\times\ 7·854$ ft. $= 914·7$ square feet, the whole of the curve surface of the saloon round the room at AmC. And $FTG - 2\,AE = 40$ ft. $-$ (5 ft. $\times$ 2) $= 30$ feet, the diameter of the ceiling MeC.

Now, by Rule 2, Prob. XIII. Superficies. $MeC^2 \times ·7854 = 30^2 \times ·7854 = 706·86$ square feet, the surface of the flat ceiling $MaCnM$.

		Square Feet.
Hence the surface of the room the spring of the arch of the saloon FAKGF	=	2513·28
The curve surface of the saloon round the room at AmC . .	=	914·70
And the surface of the flat ceiling $MaCnM$	=	706·86
The sum	=	4134·84 sq. feet,

the surface of the whole room $GKbMCmAFG$ = *Answer.*

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* Euclid, 4 p. vi. b.

GAUGING.

RULES *for reducing the* OLD MEASURE *into the* IMPERIAL, *and conversely, for reducing the* IMPERIAL MEASURE *into the* OLD.*

WINE MEASURE.

The new standard, or *Imperial* gallon, contains 277·274 cubic inches; this is its capacity at the temperature of 62 degrees of Fahrenheit's thermometer; but as the old measures were taken at 56½ degrees, it is proper, for the purpose of comparison, to reduce the imperial gallon to the same low temperature.†

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* GENERAL RULES FOR REDUCING ALL MEASURES.

FIRST. *From the Old Measure to the Imperial.*

Rule. As the number of cubic inches in the imperial standard bushel or gallon, *&c.* (when reduced to the temperature of 56½°) is to the number of cubic inches in the old standard, so is the number of bushels or gallons, *&c.* old standard, to the number of bushels or gallons, *&c.* imperial measure.

CONVERSELY.

SECOND. *From the Imperial Measure to the Old.*

Rule. As the number of cubic inches in the old standard bushel or gallon, *&c.* is to the number of cubic inches in the imperial standard bushel, gallon, *&c.* (when reduced to the temperature of 56½°), so is the number of bushels or gallons, &c. imperial standard, to the number of bushels, gallons, &c. old measure.

† The difference of temperature between 62° and 39°, when water attains its greatest density, will vary the bulk of a gal-

The imperial gallon, thus reduced, becomes 277·2 or $277\frac{1}{5}$ cubic inches; and therefore 5 imperial gallons are exactly equal to 6 wine gallons.

To reduce Wine Gallons to Imperial.

Rule. Multiply by 5, and divide by 6, or diminish the given quantity by its sixth.

CONVERSELY.

To reduce Imperial Gallons to Wine.

Rule. Multiply by 6, and divide by 5, or increase the given quantity by its fifth.

ALE AND BEER MEASURE.

The ale gallon is to the imperial gallon as 282 to $277\frac{1}{5}$; and therefore 231 ale gallons are equal exactly to 235 imperial gallons, or 58 of the former are very nearly equal to 59 of the latter.*

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lon of water rather less than $\frac{1}{3}$ of a cubic inch, or about $\frac{5}{16}$ of a cubic inch.

Then, $62° - 39° = 23$ degrees; therefore in 23° of variation of Fahrenheit a gallon of water will contract or dilate about $\frac{5}{16}$ of a cubic inch.

Now, $62° - 56\frac{1}{2}° = 5\frac{1}{2}$ degrees, the difference between the temperature at which the imperial gallon is taken, and that of the old measure.

Hence, to reduce 62° to $56\frac{1}{2}°$ will be $\frac{5}{16} \div 23 = \frac{5}{368}$ of a cubic inch, the variation of a gallon in *one* degree of temperature. $\therefore \frac{5}{368} \times 5\frac{1}{2}° =$ ·074, &c. of a cubic inch in $5\frac{1}{2}$ degrees, or in every gallon.

* This is a very near approximation; for 58 ale gallons equal $59\frac{1}{231}$ imperial gallons; and therefore the difference between the approximate and true result is only 1 in 13630.

To reduce Ale Gallons to Imperial.

Rule. 1. Multiply by 59, and divide by 58, or increase the given quantity by its 58th part.

Rule 2. Multiply by 235, and divide by 231.

CONVERSELY.

To reduce Imperial Gallons to Ale.

Rule 1. Multiply by 58, and divide by 59, or diminish the given quantity by its 59th part.

Rule 2. Multiply by 231, and divide by 235.

CORN MEASURE.

The new or imperial bushel contains 2218·192 cubic inches at 62 degrees, equals 8 imperial gallons, equals 2217·6 cubic inches when reduced to 56½ degrees; and therefore 32 new bushels equals 33 old bushels.

To reduce Old Bushels to Imperial.

Rule. Multiply by 32, and divide by 33; or increase the given quantity by its eleventh, and diminish the sum by its ninth.

CONVERSELY.

To reduce Imperial Bushels to Old.

Rule. Multiply by 33, and divide by 32; or increase the given quantity by its eighth, and diminish the sum by its twelfth.

PROBLEM I.

To find the Multipliers, Divisors, and Gauge-points belonging to the Measures now used in England.

For *square Figures*, the following multipliers and divisors are to be used.*

Divisors for Squares. Multipliers for Squares.
277·274)1·000000, &c.(·00360654 imperial gallons.
2218·192)1·000000, &c.(·00045081 imperial bushels.

For *circular Areas*, the following multipliers and divisors are to be used.

Multipliers for Circles.
277·274)·785398163397(·00283257 imperial gallons.
2218·192)·785398163397(·00035452 imperial bushels.

Divisors for Circles.
·785398163397) 277·274(353·036221 imperial gall.
·785398163397)2218·192(2824·289772 imperial bls.

The *Gauge-points* are found by extracting the square root of the divisors.

Gauge-points for Squares.
$\sqrt{277{\cdot}274} = 16{\cdot}651$ imperial gallons.
$\sqrt{2218{\cdot}192} = 47{\cdot}097$ imperial bushels.

Gauge-points for Circles.
$\sqrt{353{\cdot}036221} = 18{\cdot}789$ imperial gallons.
$\sqrt{2824{\cdot}289772} = 53{\cdot}144$ imperial bushels.

And thus the numbers in the following table were calculated.

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* *Note.* That 277·274 solid inches is an imperial gallon; and 2218·192 solid inches is an imperial bushel. So that if the solid inches in any vessel be multiplied by the respective multipliers, *viz.* ·00360654, and ·00045081, for squares: and ·00283257 and ·00035452 for circles: or divided by the respective divisors, *viz.* 277·274 and 2218·192, for squares; and 353·036221 and 2824·289772 for circles: the product, or quotient will be imperial gallons or bushels.

A TABLE of Multipliers, Divisors, and Gauge-points, for Squares and Circles.

Note. *The Areas, &c. are all in Inches.*	Multipliers for		Divisors for		Gauge-points for	
	Squares.	Circles.	Squares.	Circles.	Squares.	Circles.
Inches the area of unity . .	1·	·78539816	1·	1·2732395	1·	1·128
A superficial foot	·00694'444	·00545415	144·	183·34649	12·	13·540
A solid foot	·00057870	·00045451	1728·	2200·1579	41·569	46·905
Imperial gallon	·00360654	·00283257	277·274	353·03622	16·651	18·789
Imperial bushel	·00045081	·00035452	2218·192	2824·2897	47·097	53·144
A pound of hard soap cold	·03684598	·02893876	27·14	34·555721	5·209	5·878
A pound of hot soap	·03571428	·02804993	28·	35·650707	5·291	5·970
A pound of green soap	·03895597	·03059634	25·67	32·684059	5·066	5·716
A pound of white soft soap	·03912363	·03072723	25·56	32·544002	5·055	5·704
A pound of tallow net	·03184713	·02517191	31·4	39·979721	5·603	6·322
A pound of green starch . .	·02873563	·02256891	34·8	44·308736	5·899	6·656
A pound of dry starch	·02481389	·01937712	40·3	51·311553	6·348	7·163
A pound of flint glass	·09469696	·07437482	10·56	13·445409	3·249	3·666
A pound of white glass . . .	·07112375	·05586046	14·06	17·901747	3·749	4·230
A pound of green glass . .	·08210180	·06448260	12·18	15·508057	3·489	3·938

PROBLEM II.

To find the Area, in Ale or Wine Gallons, of any rectilineal plane Figure.

Ex. 2. By Rule, Prob. I. Superficies. $40^2 = 1600$ sq. inches, the area of the square. (*See* the Note to Rule, Prob. II. Gauging.) Then, 1600 ÷ 282 = 5·67 ale gallons. Or, by Rule 1, ale and beer measure, *of this Key*. (5·67 ÷ 58) + 5·67 = 5·76 imperial gallons = *Answer.*

Ex. 3. By Rule, Prob. II. Superficies. 50 in. × 30 in. = 1500 sq. inches, the area of the parallelogram. Then, 1500 ÷ 282 = 5·319 ale gallons. Or, by Rule 1, ale and beer measure, *of this Key.* (5·319 ÷ 58) + 5·319 = 5·41 imperial gallons = *Answer.*

Ex. 4. By Rule, Prob. III. Superficies. 40 in. × 37 in. = 1480 sq. inches, the area of the rhombus. Then, 1480 ÷ 282 = 5·24 ale gallons. Or, by Rule 1, ale and beer measure, *of this Key.* (5·24 ÷ 58) + 5·24 = 5·33 imperial gallons = *Answer.*

Ex. 5. By Rule, Prob. III. Superficies. 48 in. × 32 in. = 1536 sq. inches, the area of the rhomboides. Then, 1536 ÷ 231 = 6·649 wine gallons. Or, by the Rule, wine measure, *of this Key.* 6·649 — (6·649 ÷ 6) = 5·541 imperial gallons = *Answer.*

Ex. 6. By Rule, Prob. IV. Superficies. ½ (60 in. × 23·5 in.) = 705 sq. inches, the area of the triangle. Then, 705 ÷ 282 = 2·5 ale gallons. Or, by Rule 1, ale and beer measure, *of this Key.* (2·5 ÷ 58) + 2·5 = 2·54 imperial gallons = *Answer.*

Ex. 7. By Rule, Problem VII. Superficies. $\frac{1}{2}(\overline{15\text{ in.} + 27\text{ in.}} \times 60\text{ in.}) = 1260$ sq. inches, the area of the trapezium. Then, 1260 ÷ 282 = 4·468 ale gallons. Or, by Rule 1, ale and beer measure, *of this Key*. (4·468 ÷ 58) + 4·468 = 4·545 imperial gallons = *Answer.*

Ex. 8. By Rule 2, Prob. X. and Table. Superficies. 50^2 × 1·720477 *(tab. area.)* = 4301·1925 sq. inches, the area of the pentagon. Then, 4301·1925 ÷ 282 = 15·2524 ale gallons. Or, by Rule 1, ale and beer measure, *of this Key*. (15·2524 ÷ 58) + 15·2524 = 15·5153 imperial gallons = *Answer.*

Ex. 9. By Rule 1, Prob. X. Superficies. ½ (64 in. × 6 × 55·42 in.) = 10640·64 sq. inches, the area of the hexagon. Then, 10640·64 ÷ 282 = 37·73 ale gallons. And 10640·64 ÷ 231 = 46·06 wine gallons. Or, by the Rule, wine measure, *of this Key*. 46·06 — (46·06 ÷ 6) = 38·39 imperial gallons = 1*st Answer.*

Then, 10640·64 ÷ 2150·42 = 4·94 malt bushels. Or, by the Rule, corn measure, *of this Key*. $(4{\cdot}94 \div 11) + (4{\cdot}94 - \overline{5{\cdot}38 \div 9})$ = 4·79 imperial bushels = 2*nd Answer.*

PROBLEM III.

The Diameter of a Circle being given in Inches, to find the Area of it in Ale or Wine Gallons, &c.

Ex. 2. 45^2 × ·002785 = 5·639625 ale gallons. Or, by Rule 1, ale and beer measure, *of this Key*. (5·639625 ÷ 58) + 5·639625 = 5·736859 imperial gallons = *Answer.*

Ex. 3. 68^2 × ·002785 = 12·87784 ale gallons. Then, 68^2 × ·003399 = 15·716976 wine gallons. Or, by the Rule, wine measure, *of this Key*. 15·716976 — (15·716976 ÷ 6) = 13·09748 imperial gallons = 1*st Answer.*

Again. $68^2 \times {\cdot}000365$ (*tab. area*) $= 1{\cdot}68776$ malt bushels. Or, by the Rule, corn measure, *of this Key.* $(1{\cdot}68776 \div 11) + (1{\cdot}68776 - \overline{1{\cdot}84119 \div 9}) = 1{\cdot}63662$ imperial bushels = *2nd Answer.*

PROBLEM IV.

The Transverse, or longer diameter, and the Conjugate, or shorter Diameter, of an Ellipse being given, to find its Area in Ale or Wine Gallons.

Ex. 2. 72 in. × 50 in. × ·002785 = 10·026 ale gallons. Or, by Rule 1, ale and beer measure, *of this Key.* (10·026 ÷ 58) + 10·026 = 10·198 imperial gallons = *Answer.*

Ex. 3. 70 in. × 50 in. × ·002785 = 9·7475 ale gallons. And 70 in. × 50 in. × ·0034 = 11·9 wine gallons. Or, by the Rule, wine measure, *of this Key.* 11·9 − (11·9 ÷ 6) = 10 imperial gallons = 1*st Answer.*

Then, 70 in. × 50 in. × ·000365 (*tab. area*) = 1·2775 malt bushels. Or, by the Rule, corn measure, *of this Key.* $(1{\cdot}2775 \div 11) + (1{\cdot}2775 - \overline{1{\cdot}3936 \div 9}) = 1{\cdot}2388$ imperial bushels = *2nd Answer.*

PROBLEM V.

To find the Content of a Prism in Ale or Wine Gallons.

Ex. 2. By Rule, Prob. II. Solids. 40^2 in. × 10 in. = 16000 cubic inches, the solidity of the prism; hence, 16000 ÷ 282 = 56·7 ale gallons. Or, by Rule 1, ale and beer measure, *of this Key.* (56·7 ÷ 58) + 56·7 = 57·6 imperial gallons = *Answer.*

Ex. 3. By Rule, Prob. II. Solids. 72 in. × 33 in. × 82 in. = 194832 cubic inches, the solidity of the rectangular parallelopipedon. Hence, 194832 ÷ 282 = 690·89 ale gallons. Then, 194832 ÷ 231 = 843·42 wine gallons. Or, by the Rule, wine measure, *of this Key.* 843·42 — (843·42 ÷ 6) = 702·85 imperial gallons = 1*st Answer.*

Again. 194832 ÷ 2150·42 = 90·601 malt bushels. Or, by the Rule, corn measure, *of this Key.* $(90{\cdot}601 \div 11) + (90{\cdot}601 - \overline{98{\cdot}837 \div 9)}$ = 87·856 imperial bushels = 2*nd Answer.*

Ex. 4. By Rule 2, Prob. XIII. Superficies. $32^2 \times {\cdot}7854$ = 804·2496 sq. inches, the area of the base of the cylindrical vessel. Then, by Rule, Prob. IV. Solids. 804·2496 × 45·5 = 36593·3568, cubic inches, the solidity of the cylindrical vessel. Hence, 36593·3568 ÷ 282 = 129·763676 ale gallons. And 36593·3568 ÷ 231 = 158·4128 wine gallons. Or, by the Rule, wine measure, *of this Key.* 158·4128 — (158·4128 ÷ 6) = 132·0107 imperial gallons = 1*st Answer.*

Again. 36593·3568 ÷ 2150·42 = 17·01 malt bushels. Or, by the Rule, corn measure, *of this Key.* $(17{\cdot}01 \div 11) + (17{\cdot}01 - \overline{18{\cdot}55 \div 9)}$ = 16·49 imperial bushels = 2*nd Answer.*

PROBLEM VI.

To find the Content of any Vessel, whereof the Ends are Squares, or Rectangles of any Dimensions.

Ex. 2. 27 in. + 13·8 in. = 40·8 in. the sum of the lengths of the two ends of the frustum of the square pyramid

13·8 in. + 27 in. = 40·8 in., the sum of the breadths of the two ends of the frustum of a square pyramid.

Then, 27^2 = 729 sq. inches, the area of the greater end of the frustum.

And $13·8^2$ = 190·44 sq. inches, the area of the less end of the frustum.

Then, 40·8 in. × 40·8 in. = 1664·64 sq. inches, the product of the sum of the lengths and breadths of the two ends.

And 1664·64 + 729 + 190·44 = 2584·08 sq. inches, the sum of the areas; which × by (21 in. ÷ 6) = 9044·28 cubic inches, the solidity of the frustum of a square pyramid.

Then, 9044·28 ÷ 282 = 32·071 ale gallons. Or, by Rule 1, ale and beer measure, *of this Key*. (32·071 ÷ 58) + 32·071 = 32·623 imperial gallons = *Answer*.

Ex. 3. Here 36 in. + 48 in. = 84 in. the sum of the lengths of the two ends of the rectangular tun.

And 32 in. + 40 in. = 72 inches, the sum of the breadths of the two ends of the rectangular tun.

Then, 48 in. × 40 in. = 1920 sq. inches, the area of the greater end of the tun.

And 36 in. × 32 in. = 1152 sq. inches, the area of the less end of the tun.

Then, 84 in. × 72 in. = 6048 sq. inches, the product of the sum of the lengths and breadths of the two ends; and 6048 + 1920 + 1152 = 9120 sq. inches, the sum of the areas; which × by (60 in. ÷ 6) = 91200 cubic inches, the solidity of the rectangular tun.

Then, 91200 ÷ 282 = 323·404 ale gallons. Hence, 91200 ÷ 231 = 394·805 wine gallons. Or, by the Rule, wine measure, *of this Key*. 394·805 — (394·805 ÷ 6) = 329·005 imperial gallons = 1*st Answer*.

Again. 91200 ÷ 2150·42 = 42·41 malt bushels.

Or, by the Rule, corn measure, *of this Key.* $(42{\cdot}41 \div 11) + (42{\cdot}41 - \overline{46{\cdot}26 \div 9}) =$ 41·12 imperial bushels = *2nd Answer.*

PROBLEM VII.

To find the Content of a Vessel, in the Form of the Frustum of a Cone.

Ex. 2. $(38\text{in.} \times 20{\cdot}2\text{in.} \times 3) + \overline{38 - 20{\cdot}2}|^2 =$ 2619·64, the sum; and $2619{\cdot}64 \times (21\text{ in.} \div 3) =$ 18337·48 cubic inches, the solidity of the conical frustum. Then, $18337{\cdot}48 \div 359{\cdot}05 = 51{\cdot}07$ ale gallons. Or, by Rule 1, ale and beer measure, *of this Key.* $(51{\cdot}07 \div 58) + 51{\cdot}07 = 51{\cdot}95$ imperial gallons = *Answer.*

Ex. 3. $(40\text{in.} \times 22\text{ in.} \times 3) + \overline{40 - 22}|^2 =$ 2964, the sum; and $2964 \times (60\text{ in.} \div 3) = 59280$ cubic inches, the solidity of the conical frustum. Then, $59280 \div 359{\cdot}05 = 165{\cdot}102$ ale gallons; and $(59280 \div 294{\cdot}12) = 201{\cdot}55$ wine gallons. Or, by the Rule, wine measure, *of this Key.* $201{\cdot}55 - (201{\cdot}55 \div 6) = 167{\cdot}96$ imperial gallons = 1*st Answer.*

Again. $59280 \div 2738 = 21{\cdot}65$ malt bushels. Or, by the Rule, corn measure, *of this Key.* $(21{\cdot}65 \div 11) + (21{\cdot}65 - \overline{23{\cdot}61 \div 9}) = 20{\cdot}99$ imperial bushels = *2nd Answer.*

PROBLEM VIII.

To gauge and inch a Tun in the Form of a Frustum of a Cone, and to make an Allowance for the Drip or Fall.

Dry In.	B.	F.	G.	
	14	1 2	6·320 2·465	= 490·82 gallons when full = $\frac{19\cdot465}{8\cdot5}$
1	13	3 2	3·855 2·465	
2	13	1 2	1·390 2·465	
3	12	2 2	7·425 2·465	
4	12	0 2	4·960 2·465	
5	11	2 2	2·495 2·465	
6	11	0 2	0·030 2·465	
7	10	1 2	6·065 2·465	
8	9	3 2	3·600 2·465	
9	9	1 2	1·135 2·465	
10	8	2 2	7·170 0·250	= $\frac{17\cdot250}{8\cdot5}$
11	8	0 2	6·920 0·250	
12	7	2 2	6·670 0·250	
13	7	0 2	6·420 0·250	
14	6	2 2	6·170 0·250	
15	6	0 2	5·920 0·250	
16	5	2 2	5·670 0·250	
17	5	0 2	5·420 0·250	
18	4	2 2	5·170 0·250	
19	4	0 2	4·920 0·250	
20	3	2 1	4·670 6·958	= $\frac{15\cdot458}{8\cdot5}$
21	3	0 1	6·212 6·958	
22	2	2 1	7·754 6·958	
23	2	1 1	0·796 6·958	
24	1	3 1	2·338 6·958	
25	1	1 1	3·880 6·958	
26	·	3	5·422	left in the drip.

PROBLEM IX.

To gauge a Copper.

Dry In.	B.	F.	G.	
	23	1	4·3660	= 794·866 Gallons when full.
		2	8·2945	= $\frac{25·2945}{8·5}$
1	22	2	4·5715	
		2	8·2945	
2	21	3	4·7770	
		2	8·2945	
3	21	0	4·9825	
		2	8·2945	
4	20	1	5·1880	
		2	8·2945	
5	19	2	5·3935	
		2	8·2945	
6	18	3	5·5990	
		2	5·6095	= $\frac{22·6095}{8·5}$
7	18	0	8·4895	
		2	5·6095	
8	17	2	2·8800	
		2	5·6095	
9	16	3	5·7705	
		2	5·6095	

Dry In.	B.	F.	G.	
10	16	1	0·1610	
		2	5·6095	
11	15	2	3·0515	
		2	5·6095	
12	14	3	5·9420	
		2	3·1223	= $\frac{20·1223}{8·5}$
13	14	1	2·8197	
		2	3·1223	
14	13	2	8·1974	
		2	3·1223	
15	13	0	5·0751	
		2	3·1223	
16	12	2	1·9528	
		2	3·1223	
17	11	3	7·3305	
		2	3·1223	
18	11	1	4·2082	
		2	0·8246	= $\frac{17·8246}{8·5}$
19	10	3	3·3836	
		2	0·8246	
20	10	1	2·5590	
		2	0·8246	

Dry In.	B.	F.	G.	
21	9	3	1·7344	
		2	0·8246	
22	9	1	0·9098	
		2	0·8246	
23	8	3	0·0852	
		2	0·8246	
24	8	0	7·7606	
		1	7·2499	$= \frac{15·7499}{8·5}$
25	7	3	0·5107	
		1	7·2499	
26	7	1	1·7608	
		1	7·2499	
27	6	3	3·0109	
		1	7·2499	
28	6	1	4·2610	
		1	7·2499	
29	5	3	5·5111	
		1	7·2499	
30	5	1	6·7612	
		1	5·3426	$= \frac{13·8426}{8·5}$
31	5	0	1·4186	
		1	5·3426	

Dry In.	B.	F.	G.	
32	4	2	4·5760	
		1	5·3426	
33	4	0	7·7334	
		1	5·3426	
34	3	3	2·3908	
		1	5·3426	
35	3	1	5·5482	
		1	5·3426	
36	3	0	0·2056	
		1	3·6310	$= \frac{12·1310}{8·5}$
37	2	2	5·0746	
		1	3·6310	
38	2	1	1.4436	
		1	3·6310	
39	1	3	6·3126	
		1	3·6310	
40	1	2	2·6816	
		1	3·6310	
41	1	0	7·5506	
		1	3·6310	
42		3	3·9196	left in the crown.

PROBLEM X.

To find the Content of a Cask by the mean Diameter.

Ex. 2. *The first Variety.* $(\overline{26{\cdot}5\text{ in.} - 23\text{ in.}} \times {\cdot}68) + 23\text{ in.} = 25{\cdot}38$ inches, the mean diameter. Then, $25{\cdot}38^2 \times 28{\cdot}3\text{ in.} = 18229{\cdot}28652$ cubic inches, the solidity of the cask. Hence, $18229{\cdot}28652 \div 359{\cdot}05 = 50{\cdot}77$ ale gallons. Or, by Rule 1, ale and beer measure, *of this Key.* $(50{\cdot}77 \div 58) + 50{\cdot}77 = 51{\cdot}64$ imperial gallons = 1*st Answer.*

The second Variety. $(\overline{26{\cdot}5\text{ in.} - 23\text{ in.}} \times {\cdot}62) + 23\text{ in.} = 25{\cdot}17$ inches, the mean diameter. Then, $25{\cdot}17^2 \times 28{\cdot}3\text{ in.} = 17928{\cdot}86787$ cubic inches, the solidity of the cask. Hence, $17928{\cdot}86787 \div 359{\cdot}05 = 49{\cdot}934$ ale gallons. Or, by Rule 1, ale and beer measure, *of this Key.* $(49{\cdot}934 \div 58) + 49{\cdot}934 = 50{\cdot}794$ imperial gallons = 2*nd Answer.*

The third Variety. $(\overline{26{\cdot}5\text{ in.} - 23\text{ in.}} \times {\cdot}55) + 23\text{ in.} = 24{\cdot}925$ inches, the mean diameter. Then, $24{\cdot}925^2 \times 28{\cdot}3 = 17581{\cdot}5341875$ cubic inches, the solidity of the cask; hence, $17581{\cdot}5341875 \div 359{\cdot}05 = 48{\cdot}96681$ ale gallons. Or, by Rule 1, ale and beer measure, *of this Key.* $(48{\cdot}96681 \div 58) + 48{\cdot}96681 = 49{\cdot}81106$ imperial gallons = 3*rd Answer.*

The fourth Variety. $(\overline{26{\cdot}5\text{ in.} - 23\text{ in.}} \times {\cdot}5) + 23\text{ in.} = 24{\cdot}75$ inches, the mean diameter. Then, $24{\cdot}75^2 \times 28{\cdot}3\text{ in.} = 17335{\cdot}51875$ cubic inches, the solidity of the cask; hence, $17335{\cdot}51875 \div 359{\cdot}05 = 48{\cdot}281$ ale gallons. Or, by Rule 1, ale and beer measure, *of this Key.* $(48{\cdot}281 \div 58) + 48{\cdot}281 = 49{\cdot}113$ imperial gallons = 4*th Answer.*

To find the Content of ANY *Cask.*

Ex. 2. $(31{\cdot}5^2 \times 39) + (24{\cdot}5^2 \times 25) + (24{\cdot}5 \times 31{\cdot}5 \times 26) = 73769{\cdot}5$, the sum; and, $73769{\cdot}5 \times 42 \times {\cdot}00034 = 1053{\cdot}42846$;* which ÷ 11 = 95·76622 ale gallons. Then, 1053·42846 ÷ 9 = 117·0476 wine gallons. Or, by the *Note* to this Prob. *of this Key.* 1053·42846 ÷ by 10·8 = 97·53967 imperial gallons = *Answer.*

Ex. 3. $(32^2 \times 39) + (24^2 \times 25) + (24 \times 32 \times 26) = 74304$, the sum; and $74304 \times 40 \times {\cdot}00034 = 1010{\cdot}5344$; which ÷ by 11 = 91·8667 ale gallons. Then, 1010·5344 ÷ by 9 = 112·2816 wine gallons. Or, by the *Note* to this Prob. *of this Key.* 1010·5344 ÷ by 10·8 = 93·568 imperial gallons = *Answer.*

Ex. 4. $(48^2 \times 39) + (35{\cdot}8^2 \times 25) + (35{\cdot}8 \times 48 \times 26) = 166575{\cdot}40$, the sum; and $16675{\cdot}40 \times 55 \times {\cdot}00034 = 3114{\cdot}95998$; which ÷ by 11 = 283·17818 ale gallons. Then, 3114·95998 ÷ by 9 = 346·10666 wine gallons. Or, by the *Note* to this Prob. *of this Key.* 3114·95998 ÷ by 10·8 = 288·4222 imperial gallons = *Answer.*

Ex. 5. $(33{\cdot}8^2 \times 39) + (28{\cdot}2^2 \times 25) + (33{\cdot}8 \times 28{\cdot}2 \times 26) = 89218{\cdot}32$, the sum; and $89218{\cdot}32 \times 48 \times {\cdot}00034 = 1456{\cdot}0429824$; which ÷ by 11 = 132·3675438 ale gallons. Then, 1456·0429824 ÷ by 9 = 161·7825536 wine gallons. Or, by the *Note* to this Prob. *of this Key.* 1456·0429824 ÷ by 10·8 = 134·818794 imperial gallons = *Answer.*

PROBLEM XI.

To ullage a lying Cask.

Ex. 2. By Rule, Prob. XVII. Exam. 1. Superficies. 11 ÷ by 25 in. = ·440, the tabular height; against which stands the *Area Seg.* ·332843; which × by 48·3 = 16·0763169; and 16·0763169 ÷ ·7854 = 20·468 ale gallons. Or, by Rule 1, ale and beer measure, *of this Key.* (20·468 ÷ 58) + 20·468 = 20·82 imperial gallons by calculation = 1*st Answer.*

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* *Note.* To reduce the result to imperial gallons, divide by 10·8 instead of dividing by 9 or 11 for wine or ale gallons.

By the sliding Rule.

$\text{As } \overset{\text{on C.}}{25} : \overset{\text{on SL.}}{100} :: \overset{\text{on C.}}{11} : \overset{\text{on SL.}}{41}$

$\text{As } \overset{\text{on A.}}{100} : \overset{\text{on B.}}{48{\cdot}3} :: \overset{\text{on A.}}{41} : \overset{\text{on B.}}{19{\cdot}9}$ ale gallons = 2*nd Ans.*

Ex. 3. By Rule, Prob. XVII. Exam. 1. Superficies 8 ÷ by 32 in. = ·250, the tabular height; against which stands the *Area Seg.* ·153546, which × by 91·6 = 14·0648136, and 14·0648136 ÷ ·7854 = 17·907 gallons by calculation = 1*st Answer.*

By the Sliding Rule.

$\text{As } \overset{\text{on C.}}{32} : \overset{\text{on SL.}}{100} :: \overset{\text{on C.}}{8} : \overset{\text{on SL.}}{17{\cdot}5}$

$\text{As } \overset{\text{on A.}}{100} : \overset{\text{on B.}}{91{\cdot}6} :: \overset{\text{on A.}}{17{\cdot}5} : \overset{\text{on B.}}{16}$ gallons = 2*nd Answer.*

To ullage a standing Cask.

Ex. 2. $(EF^2 - CD^2) \times \overline{SH - IH}^2 \div IH^2 = (35^2 - 28{\cdot}7^2) \times \overline{30 - 20}^2 \div 20^2 = 100{\cdot}3275$, the *quotient.* Then, $(EF^2 - \overline{100{\cdot}3275 \div 3}) \times (SH - IH) = (35^2 - \overline{100{\cdot}3275 \div 3}) \times (30 - 20) = 11915{\cdot}575$ cubic inches = *ad*FE*a*.

Then, $\frac{11915{\cdot}575}{359{\cdot}05} = 33{\cdot}18$ ale gallons; or, by Rule 1, ale and beer measure, *of this Key.* (33·18 ÷ 58) + 33·18 = 33·75 imperial gallons, above half the content of the cask, *viz.* in *ad*FE.

Then, by the Question. The whole content of the cask ABDCA = 121·5 ale gallons, or = 148·5 wine gallons. Hence, $ad\mathrm{FE} + \frac{1}{2}\,\mathrm{ABDCA} = 33{\cdot}18 + (121{\cdot}5 \div 2) = 93{\cdot}93$ ale gallons $= ad\mathrm{DC}a$, and $\frac{11915{\cdot}575}{294{\cdot}12} + (148{\cdot}5 \div 2) = 114{\cdot}76$ wine gallons $= ad\mathrm{DC}a$, or $33{\cdot}75 + \frac{1}{2}\,(148{\cdot}5 - \overline{148{\cdot}5 \div 6}) = 95{\cdot}62$ imperial gallons $= ad\mathrm{DC}a$, the quantity of liquor in the cask by calculation = *Answer*.

Ex. 3. $(\mathrm{EF}^2 - \mathrm{CD}^2) \times \overline{\mathrm{IH} - 11}^2 \div \mathrm{IH}^2 = (26{\cdot}5^2 - 23^2) \times \overline{14{\cdot}15 - 11}^2 \div 14{\cdot}15^2 = 8{\cdot}585$, the *quotient*. Then, $(\mathrm{EF}^2 - \overline{8{\cdot}585 \div 3}) \times (\mathrm{IH} - 11) = (26{\cdot}5^2 - \overline{8{\cdot}585 \div 3}) \times (14{\cdot}15 - 11) = 2203{\cdot}07535$ cubic inches, under half the content of the cask. Then, $\frac{2203{\cdot}07535}{359{\cdot}05} = 6{\cdot}135$ ale gallons. Or, by Rule 1, ale and beer measure, *of this Key*. $(6{\cdot}135 \div 58) + 6{\cdot}135 = 6{\cdot}24$ imperial gallons, under half the content of the cask. Then, by the Question. The whole content of the cask ABDCA = 48·3 ale gallons. Hence, $(48{\cdot}3 \div 2) - 6{\cdot}135 = 18{\cdot}014$ ale gallons, or $\frac{1}{2}\,(\overline{48{\cdot}3 \div 58} + 48{\cdot}3) - 6{\cdot}24 = 18{\cdot}32$ imperial gallons, the quantity of liquor in the cask by calculation = 1*st Ans.*

By the Sliding Rule.

As $\overset{\text{on C.}}{28{\cdot}3} : \overset{\text{on SS.}}{100} :: \overset{\text{on C.}}{11} : \overset{\text{on SS.}}{37{\cdot}7}$

As $\overset{\text{on A.}}{100} : \overset{\text{on B.}}{48{\cdot}3} :: \overset{\text{on A.}}{37{\cdot}7} : \overset{\text{on B.}}{18{\cdot}3}$ ale gallons = *2nd Answer.*

LAND SURVEYING.

* * *

PROBLEM I.

To measure Offsets with a Chain and Cross-staff.

Ex. 2. By Rule, Prob. VIII. Superficies.

$\frac{1}{2}(Aa + Bb) \times AB = \frac{1}{2}(100 + 60) \times 100 = 8000$, the area of the trapezoid $= ABba$.

$\frac{1}{2}(Bb + Cc) \times BC = \frac{1}{2}(60 + 150) \times 150 = 15750$, the area of the trapezoid $= BCcb$.

$\frac{1}{2}(Cc + Dd) \times CD = \frac{1}{2}(150 + 46) \times 75 = 7350$, the area of the trapezoid $= CDdc$.

By Rule, Prob. IV. Superficies.

$\frac{1}{2}(Dd \times DE) = \frac{1}{2}(46 \times 125) = 2875$, the area of the triangle $= DEd$.

$\frac{1}{2}(EF \times fF) = \frac{1}{2}(100 \times 60) = 3000$, the area of the triangle $= EfF$.

By Rule, Prob. VIII. Superficies.

$\frac{1}{2}(fF + Gg) \times FG = \frac{1}{2}(60 + 150) \times 150 = 15750$, the area of the trapezoid $= fgGF$.

The sum $= 52725$ sq. links.

Hence, $\frac{52725}{100000} \times 4 \times 40 = 2$ roods, 4·36 perches, the area of part of the field $=$ *Answer.*

PROBLEM II.

To measure a Field in the Form of a Trapezium.

Ex. 2. By Rule, Prob. VII. Superfi. $\frac{1}{2}(CE+AF)\times DB = \frac{1}{2}(306+210)\times 592 = 152736$ square links. Hence, $\frac{152736}{100000}\times 4\times 40 =$ 1 acre, 2 roods, 4·3776 perches, the content of the field ABCD = *Answer*.

PROBLEM III.

To measure a four-sided Field, with crooked Hedges.

Ex. 1. By Rule, Prob. V. Superficies. $\frac{1}{2}(AB + BC + CA) = \frac{1}{2}(750 + 1500 + 1294) = 1772$, the half sum.

$$\left.\begin{array}{l} 1772 - 750 = 1022 \\ 1772 - 1500 = 272 \\ 1772 - 1294 = 478 \end{array}\right\}\text{the three remainders.}$$

Then, $\sqrt{1772 \times 1022 \times 272 \times 478} = \sqrt{235456895744} = 485239$ sq. links $= 4{\cdot}85239$ acres, the area of the $\triangle$ ABC.

Again. By Rule, Prob. V. Superficies. $\frac{1}{2}(AC + CD + DA) = \frac{1}{2}(1294 + 1000 + 800) = 1547$, the half sum.

$$\left.\begin{array}{l} 1547 - 1294 = 253 \\ 1547 - 1000 = 547 \\ 1547 - 800 = 747 \end{array}\right\}\text{the three remainders.}$$

Then, $\sqrt{1547 \times 253 \times 547 \times 747} = \sqrt{159925885119} = 399907$ sq. links $= 3{\cdot}99907$ acres, the area of the $\triangle$ ACD.

sq. links

By Rule, Prob. IV. Superfi. $\frac{1}{2}(300 \times 100) = 15000$

Rule, Prob. VIII. Super. $\left\{\begin{array}{l}\frac{1}{2}(100+130)\times 100 = 11500 \\ \frac{1}{2}(130+80)\times 150 = 15750\end{array}\right.$

Rule, Prob. IV. Superficies. $\frac{1}{2}(80 \times 200) = 8000$

The sum $= 50250$

sq. links $= {\cdot}5025$ acres, the area of the offsets along AB.

Sq. Links.
By Rule, Prob. IV. Super. ½ (200 × 200) = 20000
Rule, Pr. VIII. Super. ½ (200 + 150) × 500 = 87500
Rule, Prob. IV. Superficies. ½ (800 × 150) = 60000

The sum = 167500 sq.

links = 1·675 acres, the area of the offsets along BC.

Square Links.
By Rule, Prob. IV. Super. ½ (200 × 300) = 30000

$$\text{Rule, Pr. VIII. Super.}\begin{cases}\tfrac{1}{2}(200+100)\times 200 = 30000\\ \tfrac{1}{2}(100+150)\times 200 = 25000\\ \tfrac{1}{2}(150+50)\times 200 = 20000\end{cases}$$

Rule, Prob. IV. Superficies. ½ (100 × 50) = 2500

The sum = 107500 sq.

links = 1·075 acres, the area of the offsets along CD.

Square Links.
By Rule, Prob. IV. Super. ½ (200 × 150) = 15000

$$\text{Rule, Pr. VIII. Super.}\begin{cases}\tfrac{1}{2}(150+200)\times 200 = 35000\\ \tfrac{1}{2}(200+100)\times 300 = 45000\end{cases}$$

Rule, Prob. IV. Superficies. ½ (100 × 100) = 5000

The sum = 100000 sq.

links = 1 acre, the area of the offsets along DA, which is to be deducted.

Square Acres.
The area of the triangle ABC = 4·85239
The area of the triangle ACD = 3·99907
The area of the offsets along AB = ·50250
The area of the offsets along BC = 1·67500
The area of the offsets along CD = 1·07500

12·10396
The area of the offsets along DA = 1·00000

The difference = 11·10396 acres, the content of the field ABCD.

Dry In.	B.	F.	G.	
21	9	3	1·7344	
		2	0·8246	
22	9	1	0·9098	
		2	0·8246	
23	8	3	0·0852	
		2	0·8246	
24	8	0	7·7606	
		1	7·2499	$= \frac{15·7499}{8·5}$
25	7	3	0·5107	
		1	7·2499	
26	7	1	1·7608	
		1	7·2499	
27	6	3	3·0109	
		1	7·2499	
28	6	1	4·2610	
		1	7·2499	
29	5	3	5·5111	
		1	7·2499	
30	5	1	6·7612	
		1	5·3426	$= \frac{13·8426}{8·5}$
31	5	0	1·4186	
		1	5·3426	

Dry In.	B.	F.	G.	
32	4	2	4·5760	
		1	5·3426	
33	4	0	7·7334	
		1	5·3426	
34	3	3	2·3908	
		1	5·3426	
35	3	1	5·5482	
		1	5·3426	
36	3	0	0·2056	
		1	3·6310	$= \frac{12·1310}{8·5}$
37	2	2	5·0746	
		1	3·6310	
38	2	1	1.4436	
		1	3·6310	
39	1	3	6·3126	
		1	3·6310	
40	1	2	2·6816	
		1	3·6310	
41	1	0	7·5506	
		1	3·6310	
42		3	3·9196	left in the crown,

PROBLEM X.

To find the Content of a Cask by the mean Diameter.

Ex. 2. *The first Variety.* $(\overline{26\cdot5\text{ in.} - 23\text{ in.}} \times \cdot68) + 23\text{ in.} = 25\cdot38$ inches, the mean diameter. Then, $25\cdot38^2 \times 28\cdot3\text{ in.} = 18229\cdot28652$ cubic inches, the solidity of the cask. Hence, $18229\cdot28652 \div 359\cdot05 = 50\cdot77$ ale gallons. Or, by Rule 1, ale and beer measure, *of this Key.* $(50\cdot77 \div 58) + 50\cdot77 = 51\cdot64$ imperial gallons = 1*st Answer*.

The second Variety. $(\overline{26\cdot5\text{ in.} - 23\text{ in.}} \times \cdot62) + 23\text{ in.} = 25\cdot17$ inches, the mean diameter. Then, $25\cdot17^2 \times 28\cdot3\text{ in.} = 17928\cdot86787$ cubic inches, the solidity of the cask. Hence, $17928\cdot86787 \div 359\cdot05 = 49\cdot934$ ale gallons. Or, by Rule 1, ale and beer measure, *of this Key.* $(49\cdot934 \div 58) + 49\cdot934 = 50\cdot794$ imperial gallons = 2*nd Answer*.

The third Variety. $(\overline{26\cdot5\text{ in.} - 23\text{ in.}} \times \cdot55) + 23\text{ in.} = 24\cdot925$ inches, the mean diameter. Then, $24\cdot925^2 \times 28\cdot3 = 17581\cdot5341875$ cubic inches, the solidity of the cask; hence, $17581\cdot5341875 \div 359\cdot05 = 48\cdot96681$ ale gallons. Or, by Rule 1, ale and beer measure, *of this Key.* $(48\cdot96681 \div 58) + 48\cdot96681 = 49\cdot81106$ imperial gallons = 3*rd Answer*.

The fourth Variety. $(\overline{26\cdot5\text{ in.} - 23\text{ in.}} \times \cdot5) + 23\text{ in.} = 24\cdot75$ inches, the mean diameter. Then, $24\cdot75^2 \times 28\cdot3\text{ in.} = 17335\cdot51875$ cubic inches, the solidity of the cask; hence, $17335\cdot51875 \div 359\cdot05 = 48\cdot281$ ale gallons. Or, by Rule 1, ale and beer measure, *of this Key.* $(48\cdot281 \div 58) + 48\cdot281 = 49\cdot113$ imperial gallons = 4*th Answer.*

Again. By Rule, Prob. V. Superficies. $\frac{1}{2}$ (r, 3 + 3, 14 + 14, r) = $\frac{1}{2}$ (1064 + 2087·007 + 2148) = 2649·503 links, the half sum.

$$\left.\begin{array}{l} 2649{\cdot}503 - 1064{\cdot}000 = 1585{\cdot}503 \\ 2649{\cdot}503 - 2087{\cdot}007 = 562{\cdot}496 \\ 2649{\cdot}503 - 2148{\cdot}000 = 501{\cdot}503 \end{array}\right\} \text{the three remainders.}$$

Then, $\sqrt{2649{\cdot}503 \times 1585{\cdot}503 \times 562{\cdot}496 \times 501{\cdot}503}$ $= \sqrt{1185016663835{\cdot}967383923392} = 1088584{\cdot}706780$ sq. links, the area of the triangle r, 3, 14.

*$\sqrt{yW^2 + We^2} = \sqrt{1160^2 + 1676^2} = \sqrt{4154576}$ = 2038·277704 links, the hypothenuse or diagonal from e to y.

Then, by Rule, Prob. V. Superficies. $\frac{1}{2}$ (e, 14 + 14, y + y, e) = $\frac{1}{2}$ (180 + 1950 + 2038·277) = 2084·138 links, the half sum.

$$\left.\begin{array}{l} 2084{\cdot}138 - 180{\cdot}000 = 1904{\cdot}138 \\ 2084{\cdot}138 - 1950{\cdot}000 = 134{\cdot}138 \\ 2084{\cdot}138 - 2038{\cdot}277 = 45{\cdot}861 \end{array}\right\} \text{the three remainders.}$$

Then, $\sqrt{2084{\cdot}138 \times 1904{\cdot}138 \times 134{\cdot}138 \times 45{\cdot}861} = \sqrt{244129487742{\cdot}732345857992} = 156246{\cdot}435936$ sq. links, the area of the triangle e, y, 14.

Now, by Rule, Prob. IV. Superficies. $\frac{1}{2}$ ($yW \times We$) = $\frac{1}{2}$ (1160 × 1676) = 972080 sq. links, the area of the right angle triangle W, y, e.

*$\sqrt{GA^2 + Ar^2} = \sqrt{744^2 + 510^2} = \sqrt{813636} =$ 902·017738 links, the hypothenuse or diagonal from r to G.

Then, by Rule, Prob. V. Superficies. $\frac{1}{2}$ ($ry + y$G$+$ Gr) = $\frac{1}{2}$ (198+784+902·017) = 942·008 links, the half sum.

$$\left.\begin{array}{l} 942{\cdot}008 - 198{\cdot}000 = 744{\cdot}008 \\ 942{\cdot}008 - 784{\cdot}000 = 158{\cdot}008 \\ 942{\cdot}008 - 902{\cdot}017 = 39{\cdot}991 \end{array}\right\} \text{the three remainders.}$$

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* Euclid, 47 p. i. b.

Then, $\sqrt{942{\cdot}008 \times 744{\cdot}008 \times 158{\cdot}008 \times 39{\cdot}991} = \sqrt{4428672204{\cdot}742606331392} = 66548{\cdot}269735$ square links, the area of the triangle yrG.

Now, by Rule, Prob. IV. Superficies. $\frac{1}{2}(r\mathrm{A} \times \mathrm{AG}) = \frac{1}{2}(510 \times 744) = 189720$ square links, the area of the right angle triangle AGr.

By Rule, Prob. V. Superficies. $\frac{1}{2}(e, 15 + 15, \mathrm{T} + \mathrm{T}e) = \frac{1}{2}(356 + 580 + 588) = 762$ links, the half sum.

$$\left.\begin{array}{l} 762 - 356 = 406 \\ 762 - 580 = 182 \\ 762 - 588 = 174 \end{array}\right\} \text{the three remainders.}$$

Then, $\sqrt{762 \times 406 \times 182 \times 174} = \sqrt{9797192496} = 98980{\cdot}768313$ square links, the area of the triangle eT, 15.

		Sq. Links.
By Rule, Prob. VIII. Superficies.	$\frac{1}{2}(46+66) \times 380 =$	21280
	$\frac{1}{2}(66+34) \times 670 =$	33500
Rule, Prob. II. Super.	$34 \times 270 =$	9180
Rule, Prob. VIII. Superficies.	$\frac{1}{2}(86+96) \times 110 =$	10010
	$\frac{1}{2}(96+76) \times 88 =$	7568
	$\frac{1}{2}(76+84) \times 136 =$	10880
	$\frac{1}{2}(84+50) \times 330 =$	22110
	The sum =	114528

The sum = 114528 sq. links, the area of the offsets along 12, 7.

		Square Links.
Rule, Prob. IV. Super.	$\frac{1}{2}(280 \times 40) =$	5600
Rule, Pr. VIII. Super.	$\frac{1}{2}(40+30) \times 890 =$	31150
	$\frac{1}{2}(86+80) \times 360 =$	29880
	$\frac{1}{2}(80+54) \times 200 =$	13400
Rule, Prob. IV. Superficies.	$\frac{1}{2}(342 \times 54) =$	9234
	The sum =	89264

The sum = 89264 sq. links, the area of the offsets along C, 9.

Square Links.

By Rule, Prob. IV. Super. $\frac{1}{2}(330 \times 22) = 3630$

Rule, Pr. VIII. Super. $\left\{\begin{array}{l} \frac{1}{2}(22+32) \times 544 = 14688 \\ \frac{1}{2}(32+60) \times 270 = 12420 \\ \frac{1}{2}(60+48) \times 300 = 16200 \end{array}\right.$

Rule, Prob. IV. Superficies. $\frac{1}{2}(350 \times 48) = 8400$

The sum $= 55338$ sq.

links, the area of the offsets along 9, A.

Square Links.

Rule, Pr. VIII. Super. $\left\{\begin{array}{l} \frac{1}{2}(72+150) \times 120 = 13320 \\ \frac{1}{2}(150+130) \times 112 = 15680 \end{array}\right.$

Rule, Prob. IV. Superfi. $\frac{1}{2}(144 \times 130) = 9360$

The sum $= 38360$ sq.

links, the area of the offsets along GW.

Square Links.

By Rule, Prob. IV. Superfi. $\frac{1}{2}(780 \times 24) = 9360$

By Rule, Prob. VIII. Superficies. $\left\{\begin{array}{l} \frac{1}{2}(50+24) \times 308 = 11396 \\ \frac{1}{2}(76+50) \times 100 = 6300 \end{array}\right.$

Rule, Prob. II. Superficies. $76 \times 200 = 15200$

Rule, Prob. VIII. Superficies. $\left\{\begin{array}{l} \frac{1}{2}(76+40) \times 200 = 11600 \\ \frac{1}{2}(40+20) \times 80 = 2400 \end{array}\right.$

The sum $= 56256$ sq.

links, the area of the offsets along WTM.

Square Links.

By Rule, Prob. VIII. Superficies. $\left\{\begin{array}{l} \frac{1}{2}(36+60) \times 1040 = 49920 \\ \frac{1}{2}(60+90) \times 388 = 29100 \\ \frac{1}{2}(90+120) \times 448 = 47040 \\ \frac{1}{2}(120+56) \times 474 = 41712 \end{array}\right.$

By Rule, Prob. IV. Super. $\frac{1}{2}(170 \times 56) = 4760$

The sum $= 172532$ sq.

links, the area of the offsets along M*m*.

Square Links.

By Rule, Prob. IV. Superfi. $\frac{1}{2}(130 \times 60) = 3900$

By Rule, Prob. VIII. Superficies. . .
$$\left\{\begin{aligned} &\tfrac{1}{2}(160+60) \times 450 = 49500 \\ &\tfrac{1}{2}(160+40) \times 670 = 67000 \\ &\tfrac{1}{2}(40+80) \times 290 = 17400 \\ &\tfrac{1}{2}(80+20) \times 190 = 9500 \\ &\tfrac{1}{2}(20+30) \times 170 = 4250 \end{aligned}\right.$$

The sum = 151550 sq. links, the area of the offsets along *m*, 12, which is to be deducted from the whole estate.

Square Links.

The area of the trapezoid 12, B, 3, 13. = 2315040·000000
The area of the triangle BCD . . . = 3820926·001691
The area of the triangle AD, 9 . . . = 388307·368216
The area of the rt. angle triangle 13, 3, 14 = 1072000·000000
The area of the triangle *r*, 3, 14 . . . = 1088584·706780
The area of the triangle *ey*, 14 . . . = 156246·435936
The area of the rt. angle triangle *wye* . = 972080·000000
The area of the triangle *yr*G . . . . = 66548·269735
The area of the rt. angle-triangle AG*r* . = 189720·000000
The area of the triangle *s*T, 15 . . . = 98980·768313
The area of the offsets along 12, 7 . . = 114528·000000
The area of the offsets along C, 9 . . = 89264·000000
The area of the offsets along 9, A . . = 55338·000000 } offsets.
The area of the offtets along GW . . = 38360·000000
The area of the offsets along WTM . = 56256·000000
The area of the offsets along M*m* . . = 172532·000000

The sum = 10694711·550671
The area of the offsets along *m*, 12 deduct 151550·000000

The area of the whole estate = 10543161·550671 sq. links = 105 acres, 1 rood, 29 perches, the content of the whole estate, 12, C, 9, AWM*m*, 12.

MISCELLANEOUS QUESTIONS IN MENSURATION.

Ex. 1. (47 ft. 9 in. × 18 ft. 6 in.) ÷ 9 ft. = 98 sq. yards, 1 sq. foot, 4 inches, 6 seconds, the content of the pavement = *Answer*.

Ex. 2. 18^2 = 324 sq. inches = 2¼ sq. feet, the area of each stone.
Then, (21½ ft. × 17½ ft.) ÷ 2¼ ft. = 167$\frac{2}{9}$, the number of stones to pave the room = *Answer*.

Ex. 3. (109·75 ft. × 9·25 ft.) − (6·5 ft. × 5·75 ft. × 2) ÷ 11·25 ft. = 83·59 sq. yards of tapestry to hang round the room = *Answer*.

Ex. 4. Here, by Rule 1, Prob. XI. Superficies. 27·5 in. × 3·1416 = 86·394 inches, the circumference; and, by Rule 2, Prob. XII. Solids. (86·394 in. × 27·5 in.) ÷ 144 = 16·4988 square feet, the convex surface = 1*st Answer*.

Hence, by Rule 2, Prob. XII. Solids. 16·4988 × (27·5 in. × $\frac{1}{6}$) ÷ 12 = 6·301625 solid feet = 2*nd Answer*.

Ex. 5. *First*. To find the solidity of the segment, *see* the Diagram, and Rule 2, Prob. XIII. Solids. $(3\,AO^2 + OD^2) \times OD \times {\cdot}5236 = (\overline{12^2 \times 3} + 10^2) \times 10 \times {\cdot}5236 = 2785{\cdot}552$ cubic inches, the solidity of the segment ABDA = 1*st Answer*.

Second. To find the Superficies of the segment, *see* the *Note* to Rule 2, Prob. XII. Solids. By *Note* 8, Prob. XII. Superficies, or Euclid, 35 P. iii. B. ($AO^2 \div OD) + OD = (12^2 \div 10) + 10 = 24{\cdot}4$ inches, the diameter DE. Then, by Rule 2, Prob. XIII. Superficies. $AB^2 \times {\cdot}7854 = 24^2 \times {\cdot}7854 = 452{\cdot}3904$ square inches, the area of the base of the segment. Now, by Rule 1, Prob. XI. Superficies. $DE \times 3{\cdot}1416 = 24{\cdot}4 \times 3{\cdot}1416 = 76{\cdot}65504$ inches, the circumference of the sphere ADBEA. Then, by *Note* to Rule 2, Prob. XII. Solids. $ADBEA \times OD = 76{\cdot}65504 \times 10 = 766{\cdot}5504$ sq. inches, the curve surface of the segment. Hence, $766{\cdot}5504 + 452{\cdot}3904 = 1218{\cdot}9408$ sq. inches, the superficies of the whole segment = 2nd *Answer*.

Ex. 6. By Rule 1, Prob. V. Timber Measure. 18 ft. 6 in. ÷ 4 = 4 ft. 7 in. 6 seconds, the quarter girt. Then, ($(\overline{4\text{ ft. }7\text{ in. }6\text{ sec.}})^2$ × 24 ft.) ÷ 40 = 513 sol. ft. 4 in. 6 sec. ÷ 40 = 12 sol. tons, 33 sol. feet, 4 inches, 6 seconds, the content of the tree = *Answer*.

Ex. 7. (33 ft. 7 in. × 18 ft. 9 in. × 5 ft. 9 in.) ÷ 324 = 3620 sol. ft. 8 in. 5 sec. 3 thirds ÷ 324 = 11 sol. floors, 56 sol. feet, 8 inches, 5 seconds, 3 thirds, the content of the cellar = *Answer*.

Ex. 8. (48 ft. 9 in. × 35 ft. 6 in.) ÷ 100 = 1730 sq. ft. 7 in. 6 sec. ÷ 100 = 17 squares, 30 sq. feet, 7½ inches, the content of the roof = *Answer*.

Ex. 9. *To find the solidity of the whole Cone. First.* By Rule 2, Prob. XIII. Superficies. $42^2 \times {\cdot}7854 = 1385{\cdot}4456$ sq. inches, the area of the base. And, by Rule, Prob. VI. Solids. $1385{\cdot}4456 \times (94\text{ in.} \div 3) = 43410{\cdot}6288$ cubic inches, the solidity of the whole cone.

Now, $94^3 = 830584$ inches, the cube of the altitude.
And $1728 \times 2 = 3456$ cubic inches in two feet.

*$\therefore$, as 43410·6288, the solidity of the cone, : 3456, the solidity of two feet, :: 830584, the cube of altitude

of the cone : 66124·319857813 inches, the cube of the altitude of the two feet. Hence, $\sqrt[3]{66124{\cdot}319857813} = 40{\cdot}437$ inches, the altitude of the top part to be cut off = *Answer.*[*]

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* Euclid, 12 p. xii. b.

Ex. 10. *First. Find the height of the whole Pyramid.* By *Note* to Prob. VII. Solids. (*See* also the Diagram.) [†]AG − CO =(AE) : CO :: GO : OV; *viz.* as 10·5 in. − 1·5 in. : 1·5 ft. :: 12 in. : 2 feet. Hence, OV + GO = 2 ft. + 12 ft. = 14 ft. = GV, the height of the pyramid ABVA.

Second. Find its solid content. By Rule, Prob. V. Solids. $21^2 = 441$ sq. inches, the area of the base. Hence, $441 \times (14 \div 3) \div 144 = 14{\cdot}291\frac{2}{3}$ cubic feet, the solidity of the whole pyramid ABVA. Then, $GV^3 = 14^3 = 2744$ feet, the cube of the altitude of the pyramid ABVA; and ABVA − 5 ft. = $14{\cdot}291\frac{2}{3}$ − 5 ft. = $9{\cdot}291\frac{2}{3}$ cubic feet, the remainder. Then, as $14{\cdot}291\frac{2}{3}$ cubic ft. : $9{\cdot}291\frac{2}{3}$ cubic ft. :: 2744 ft. cube of alt. : 1784 feet, the cube of the length; and $\sqrt[3]{1784} = 12{\cdot}128$ feet, the length from the top, the section is to be made. Hence, 14 ft. − 12·128 ft. = 1·872 feet, the length of five solid feet at the greater end to be cut off = *Answer.*

Ex. 11. *(See Fig.* 11 *of this Key.)* By Rule 2, Prob. XIII. Superficies. $FD^2 \times {\cdot}7854 = 40^2 \times {\cdot}7854 = 1256{\cdot}64$ sq. inches, the area of the whole circular side of the grindstone DEFG.

To obtain the first Man's Share.

20*s.* : 9*s.* :: 1256·64 sq. in. : 565·488 sq. in. =

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[*] Now supposing it were required to cut off 2 solid feet from the bottom, or greater end, then it would be, from 43410·6288 cubic inches subtract 3456 cubic inches, and the remainder is 39954·6288 cubic inches. Then say, as 43410·6288 cub. in. : 39954·6288 cub. in. :: 830584 cube alti. : 764459·6801 inches, the cube of the altitude; hence $\sqrt[3]{764459{\cdot}6801} = 91{\cdot}4$ inches, which must be cut from the top, or 2·6 inches from the bottom.

[†] Euclid, 4 p. vi. b.

mm, the area between the two concentric circles CD = the first man's share. Then, the area of the circle DEFG − *mm* = 1256·64 sq. in. − 565·488 sq. in. = 691·152 sq. inches, the area of the two remaining shares CHKL. Hence, the area of the circle CHKL ÷ ·7854 = 691·152 sq. in. ÷ ·7854 = 880 in. = KC^2, the square of the diameter of the two remaining shares; and $\sqrt{KC^2} = \sqrt{880}$ = 29·6647939 inches = KC, the diameter of the two remaining shares; which ÷ by 2 = 14·8323969 in. = AC, the radius. Then, the radius AD − AC = 20 in. − 14·8323969 in. = 5·1676031 in. = CD, the first man must grind down from the radius AD = 1*st Answer.*

To obtain the second and third Man's Share.

20*s*. : 6*s*. :: 1256·64 sq. in. : 376·992 sq. in. = *nn*, the area between the two concentric circles BC = the second man's share. Then, the area of the circle CHKL − *nn* = 691·152 sq. in. − 376·992 sq. in. = 314·16 sq. inches, the area of the third man's share BMNO. Hence, the area of the circle BMNO ÷ ·7854 = 314·16 sq. in. ÷ ·7854 = 400 in. = NB^2, the square of the diameter of the third man's share; and $\sqrt{NB^2} = \sqrt{400}$ = 20 in. = NB, the diameter of the third man's share; which ÷ by 2 = 10 in. = AB, the radius of the circle BMNO, which the third man must grind away = 2*nd Answer.*

Then, the radius AC − AB = 14·8323969 in. − 10 in. = 4·8323969 in. = BC, the second man must grind down from the radius AC = 3*rd Answer.*

Or, thus:—

*All circles are to each other as the squares of their diameters; and each man must grind away a surface proportionable to the money he paid?

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* Euclid, 2 p. xii. b.

$$20s. : 5s. :: 20^2 \text{ in.} : 100 \left\{ \begin{array}{l} \text{and } \sqrt{100} = 10 \text{ in.} = AB, \\ \text{the radius of the circle} \\ \text{BMNO, which the \textit{third}} \\ \text{man must grind away} = \\ \text{1st \textit{Answer}.} \end{array} \right.$$

$$20s. : 6s. :: 20^2 \text{ in.} : 120$$

The sum = 220; and $\sqrt{220} = 14\cdot832397$ in. = AC, the radius of the circle CHKL, which *two* men must grind away.

Then, AC − AB = 14·832397 in. − 10 in. = 4·832397 in. = BC, the breadth of the ring which the *second* man must grind away = 2*nd Answer.*

Again. AD − AC = 20 in. − 14·832397 in. = 5·167603 inches, the breadth of the ring the *first* man must grind away = 3*rd Answer.*

[†]Ex. 12. (*See Fig.* 12 *of this Key.*) [*]Similar cones are to each other as the cubes of their altitude.

First, as ABG : EFG :: KG^3 : HG^3, *viz.* as 3 : 1 :: 20^3 : 2666·6′, the cube of the altitude. Hence, $\sqrt[3]{2666\cdot6'} = 13\cdot867$ inches, the altitude of the first section HG = 1*st Answer.*

Second. As ABG : CDG :: KG^3 : IG^3; *viz.* as 3 : 2 :: 20^3 : 5333·3′, the cube of the altitude. Hence, $\sqrt[3]{5333\cdot3'} = 17\cdot471$ inches, the altitude IG.

Then, IG − HG = 17·471 in. − 13·867 in. = 3·604 inches, the altitude of the second section IH = 2*nd Answer.*

Third. KG − IG = 20 in. − 17·471 in. = 2·529 inches, the altitude of the third section KI = 3*rd Answer.*

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[*] Euclid, 12 p. xii. b.

[†] In former editions, the following question is found, and has been omitted by Mr. Keith, because its solution depends on

Ex. 13. First, find three numbers in proportion to the depth, width, and length: thus, suppose the width 1, the depth will be ·7, and the length 4·2; hence the solidity will be 2·94 cubic feet. But solids are to each other as the cubes of their like parts; consequently, as 2·94 cub. ft. : 367·5 cub. ft. :: 1^3 : 125, the cube of

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fluxions; but as the ingenious and inquisitive may be gratified by its solution, it is for their satisfaction here introduced;

Ex. 12. A gard'ner he had an upright cone,
Out of which should be cut him a rolling-stone,
The biggest that e'er it could make:
The mason he said, that there was a rule
For such sort of work, but he had a thick skull;
Now help him for pity's sake.

(*See Fig.* 12 *of this Key.*) To determine the greatest cylinder EO, that can be inscribed in the given cone AGB.

Let AB $= d$, GK $= c$, EF $= y$, KH $= x$, $n = ·7854$.

Then, by the principles of mensuration, xy^2n will express the content of the cylinder EO, and this is to be a maximum by the Question; that is, xy^2 is to be a maximum, putting its fluxion $=0$; $\therefore 2\,xy\dot{y} + y^2\dot{x} = 0$, or $y\dot{x} = -2\,x\dot{y}$.

By reason of the parallels AB, EF; GK : GH :: AB : EF, whence GK × EF = GH × AB; that is, $cy = d \times (c - x)$, and take the fluxions $c\dot{y} = -d\dot{x}$, or $-\dot{y} = \frac{d\dot{x}}{c}$, and $y = \frac{d}{c} \times (c - x)$, writing these respectively for $\dot{y}$ and y, in the expression $y\dot{x} = -2\,x\dot{y}$, it becomes $\frac{d}{c} \times (c - x)\,\dot{x} = \frac{2\,dx\dot{x}}{c}$; dividing by $\frac{d\dot{x}}{c}$ $c - x = 2\,x$, or $3\,x = c$, $x = \frac{c}{3}$ and $y = \frac{d}{c} \times (c - x) = \frac{d}{c} \times \frac{2\,c}{3} = \frac{2\,d}{3}$; these are the dimensions of the greatest cylinder EO, that can be cut out of the given cone AGB. The solid content of the whole cone is $\frac{1}{3} \times d^2cn$; and the content of the greatest inscribed cylinder from the foregoing dimensions is $\frac{1}{3} \times \frac{4}{9}\,d^2cn$ $\therefore$ the content of the greatest inscribed cylinder is $\frac{4}{9}$ths of the content of the whole cone.

Those who wish to see the preceding conclusions deduced geometrically may consult " Simpson's *Geometry*, on the *Maxima* and *Minima* of Geometrical Quantities, *Theorem* 19, *page* 209, *fifth edition*.

the width. Hence, $\sqrt[3]{125} = 5$ feet, the width = 1*st Answer*.

Now, ·7 × ·5 = 3·5 feet, the depth = *2nd Answer*.

Then, 3·5 × 6 = 21 feet, the length = *3rd Answer*.

Then, (367·5 × 1728) ÷ 2150·42 = 295 bushels, 1 peck, 3·82422 pints. Or, by Rule to corn measure *of this Key*. (295 bls. 1 peck, 3·82422 pints ÷ 11) + 295 bls. 1 peck, 3·82422 pints — (322 bls. 0 peck, 9·99005 pints ÷ 9) = 286 bls. 1 peck, 7·10227 pints, imperial gallons = 4*th Answer*.

Ex. 14. By Rule 2, Prob. XIII. Superficies. $18{\cdot}5^2$ × ·7854 = 268·803150 sq. inches, the area of the base. Hence, by Rule, Prob. IV. Solids. 268·80315 × 8 = 2150·4252 cubic inches in a cylindrical bushel.

Then, 2150·4252 ÷ (7·5 × ·7854) = 365·06′ inches, the square of the diameter.

Hence, $\sqrt{365{\cdot}06'}$ = 19·1067 inches, the diameter when the depth is 7·5 inches = *Answer*.

Ex. 15. Here the area of one acre = 43560 sq. feet.

Then, 43560 sq. ft. ÷ ·7854 = 55462·1847, the square of the diameter of an acre. Hence, $\sqrt{55462{\cdot}1847}$ = 235·50 feet, the diameter of an acre, which ÷ by 2 = 117·75 feet, the radius = the length of the cord = *Answer*.

Ex. 16. $16{\cdot}5^2$ = 272·25 sq. feet, the area of the old kiln; which × by 3 = 816·75 sq. feet, three times the area of the old kiln. Hence, $\sqrt{816{\cdot}75}$ = 28·578 feet, the side of the new kiln = *Answer*.

Ex. 17. $26{\cdot}3^2$ = 691·69 inches, the square of the less cistern's diameter, which × by 2 = 1383·38 inches, twice the square of the diameter. Hence, $\sqrt{1383{\cdot}38}$ = 37·19 inches, the diameter of the greater cistern = 1*st Answer*.

Then, by Rule, Prob. III. Gauging. $37{\cdot}19^2$ × ·002785 × 52·5 in. = 202·22593852125 ale gallons. Or, by Rule 1, ale and beer measure, *of this Key*.

(202·22593852125 ÷ 58) + 202·22593852125 = 205·71259263368 imperial gallons, the content of the greater cistern = 2*nd Answer.*

Now, the square of the less cistern's diameter as above = 691·69 in. Hence, by Rule, Prob. III. Gauging. 691·69 in. × ·002785 × 52·5 in. = 101·133724125 ale gallons; or, by Rule 1, ale and beer measure, *of this Key:* (101·133724125 ÷ 58) + 101·133724125 = 102·877409023 imperial gallons, the content of the less cistern = 3*rd Answer.*

Ex. 18. By the General Rule, Prob. X. Gauging. (32^2 × 39) + (25^2 × 25) + (25 × 32 × 26) = 76361, the sum; and 76361 × 40 × ·00034 = 1038·50960, which ÷ by 11 = 94·40996 ale gallons. Or, by *Note* to Prob. X. Gauging, *of this Key.* 1038·50960 ÷ 10·8 = 96·1582 imperial gallons, the content of the cask = *Answer.*

Ex. 19. Here 20^3 = 8000 inches, the length cubed. *As 217 lbs. : 1000 lbs. :: 8000 cubic in. : 36866·359447004in., the cube of the length. Hence, $\sqrt[3]{36866\cdot359447004}$ = 33·282 inches, the length of the stone weighing 1000 pounds = 1*st Answer.*

Then say, as 20 in. : 33·282 in. :: 15 in. : 24·961, inches, the breadth of the stone weighing 1000 pounds = 2*nd Answer.*

And, as 20 in. : 33·282 in. :: 8 in. : 13·312 in., the thickness of the stone weighing 1000 pounds = 3*rd Answer.*

Ex. 20. Here 4^3 = 64 inches, the cube of the diameter of the less bullet; and 9^3 = 729 inches, the cube of the diameter of the greater bullet. Then, by the Rule, Prob. XXXVII. Solids. †As 64 cub. in. : 729 cub. in. :: 9 lbs. : 102·515 pounds, the weight of the bullet = *Answer.*

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* Euclid, 33 p. xi. b.

† Euclid, 18 p. xii. b.

Ex. 21. Here, $5^3 = 125$ inches, the cube of the side of the less pyramid; and $30^3 = 27000$ inches, the cube of the side of the greater pyramid.

*As 125 cub. in. : 27000 cub. in. :: 12·25 lbs. : 2646 pounds, the weight of the sq. pyramid = *Answer*.

Ex. 22. Here, $6^3 = 216$ inches, the diameter cubed. Then, as 11 lbs. : 500 lbs. :: 216 cub. in. : 9818·1'8' in., the cube of the diameter. Hence, $\sqrt[3]{9818{\cdot}1'8'} = 21{\cdot}4$ inches, the diameter of the globe = *Answer*.

Ex. 23. By Rule 3, Prob. VII. Solids. $(21^2 + 9^2) + (21 \times 9) = 711$ inches, the sum. Then, 711 in. × (4·828427 *(tab. area.)* ÷ 3) × 15 ft. = 17165·050875; which ÷ by 144 = 119·201742 feet, the solidity of the frustum = *Answer*.

Ex. 24. *(See Fig. 9 of this Key.)* *First.* Find the perpendicular height of the cone. †AD − FI = (AO) : AD :: DI : DC; *viz.* as 18 in. − 10 in. : 18 in. :: 215 in. : 483·75 inches, the perpendicular height of the whole cone ABCA.

Second. Find the solidity of the whole cone.. By Rule, Prob. VI. Solids. $AB^2 \times {\cdot}7854 = 36^2 \times {\cdot}7854 = 1017{\cdot}8784$ sq. inches, the area of the base AGBHA. Hence, AGBHA × ⅓DC = 1017·8784 sq. in. × (483·75 in. ÷ 3) = 164132·892 cubic inches; which ÷ by 1728 = 94·9843 cubic feet, the solidity of the whole cone ABCA = *1st Answer*.

Third. To find the solidity of the frustum. By Rule 2, Prob. VIII. Solids. $(AB \times FK) + AB^2 + FK^2 = (36 \times 20) + 36^2 + 20^2 = 2416$ sq. inches. Then, 2416 sq. in. × 215 in. × (·7854 ÷ 3) = 135989·392 cubic inches; which ÷ by 1728 = 78·6975 cubic feet, the solidity of the frustum ABKFA = *2nd Answer*.

Ex. 25. *(See Fig. 9 of this Key.)* $IC^3 = 200^3 = 8000000$ inches, the cube of the altitude of the top part.

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* Euclid, 8 p. xii. b.

† Euclid, 4 p. vi. b.

Now, $ABKFA + FKCF = 159610$ cub. in. $+ 26171$ cub. in. $= 185781$ cubic inches, the solidity of the whole cone ABCA.

Then, $FKCF : ABCA :: IC^3 : DC^3$, *viz.* as 26171 cub. in. : 185781 cub. in. :: 8000000 in. cube of alti. : 56789881·93038095602002 inches, the cube of the altitude of the whole cone.

Hence, $\sqrt[3]{56789881{\cdot}93038095602002} = 384{\cdot}3766$ in. $= DC$, the perpendicular height of the whole cone $ABCA =$ 1*st Answer*.

To get the area and diameter of the greater base AGBHA. Here $3\,(ABCA \div DC) = (185781 \text{ cub. in.} \div 384{\cdot}3766 \text{ in.}) \times 3 = 1449{\cdot}9918$ sq. inches, the area of the greater base AGBHA.

Then, $AGBHA \div {\cdot}7854 = 1449{\cdot}9918 \text{ sq. in.} \div {\cdot}7854 = 1846{\cdot}18258212 \text{ sq. in.} = AB^2$, the square of the diameter. Hence, $\sqrt{AB^2} = \sqrt{1846{\cdot}18258212} = 42{\cdot}9672$ inches, the diameter $AB =$ 2*nd Answer*.

Again. To get the area and diameter of the less base. Here $3\,(FKCF \div IC) = (26171 \text{ cub. in.} \div 200 \text{ in.}) \times 3 = 392{\cdot}565$ sq. inches, the area of the less base FPKLF.

Then, $FPKLF \div {\cdot}7854 = 392{\cdot}565 \text{ sq. in.} \div {\cdot}7854 = 499{\cdot}82811306 \text{ sq. in.} = FK^2$, the square of the diameter, Hence, $\sqrt{FK^2} = \sqrt{499{\cdot}82811306} = 22{\cdot}3568$ inches. the diameter $FK =$ 3*rd Answer*.

Ex. 26. *First.* Find the content of a conical frustum, whose diameters are 5 and 2, and depth 12 feet, By Rule 2, Prob. VIII. Solids. $(5 \times 2) + (5^2 + 2^2) \times 12 \text{ ft.} \times ({\cdot}7854 \div 3) = 122{\cdot}5224$ cubic feet, the solidity of the conical frustum, whose diameters are 5 ft. and 2 feet.

Then, as 122·5224 cub. ft. : 20 cub. ft. :: 25 sq. feet, (the greater diameter squared) : 4·080886 ft., the square of the greater diameter. Hence, $\sqrt{4{\cdot}080886} = 2{\cdot}02012$ feet, the greater diameter = 1*st Answer*.

Now, as 5 ft. : 2 ft. :: 2·02012 ft. : ·80804 feet, the less diameter = 2*nd Answer*.

Ex. 27.

$$
\begin{array}{rcl}
\text{Feet. In.}\ \ \text{Feet. In.} & & \text{Sq. Feet. In. Sec.}\\
129.6 \times 16.9 & = & 2169.1.6 \text{ content of the room.}\\
7.3 \times 4.6 \times 2 = & & \left.\begin{array}{l}65.3.0 \text{ at work}\\ 32.7.6 \text{ at half do.}\end{array}\right\} \text{the shutters}\\
7.0 \times 3.3 & = & \left.\begin{array}{l}22.9.0 \text{ at work}\\ 11.4.6 \text{ at half do.}\end{array}\right\} \text{the door.}\\
24.6 \times 1.9 \times 2 & = & 85.9.0 \text{ cheek boards.}\\
\hline
\text{The sum} & = & 2386.10.6\\
\left.\begin{array}{l}\text{Door-case, 3 ft.}\\ \text{6 in.} \times \text{7 ft. 0 in.}\\ \text{2 window-lights,}\\ \text{7 ft. 3 in.} \times \text{5 ft.}\\ \text{6 in.} \times 2.\end{array}\right\} & = & 104.3.0 \text{ which deduct.}\\
\hline
 & & 9)2282.7.6\\
\hline
 & & 253 \text{ sq. yds. 5 sq. ft. 7 in. 6 sec.}
\end{array}
$$

= *Answer.*

Ex. 28. Assume the breadth 1 foot; then the height must be 5 feet, and length 40 feet. Hence, 1 ft. × 5 ft. × 40 ft. = 200 feet, the solidity of the assumed cube.

Then say, as 200 cub. ft. : 18225 cub. ft. :: 1 ft. (the cube of the breadth of the assumed cube) : 91·125 feet, the cube of the breadth of the actual cube. Hence, $\sqrt[3]{91{\cdot}125} = 4{\cdot}5$ feet, the breadth = 1*st Answer.*

∴ 4·5 ft. × 5 = 22·5 feet, the height = 2*nd Ans.* And ∴ 22·5 ft. × 8 = 180 feet, the length = 3*rd Ans.*

Ex. 29. *(See Fig.* 13 *of this Key.)* [*]$\sqrt{AC^2 - AB^2} = \sqrt{39^2 - 15^2} = \sqrt{1296} = 36$ ft. = BC, the length of the piece standing.

∴ BC + AC = 36 ft. + 39 ft. = 75 feet, the height of the maypole when standing = *Answer.*

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[*] Euclid, 47 p. i. b.

Ex. 30. (*See Fig.* 14 *of this Key.*) *As AB : BC :: BE : CD; *viz.* as 4·1 ft. : 65 ft. :: 5 ft. : 79·26 feet, the height of the maypole = *Answer.*

Ex. 31. If the diameter of a sphere be 1, the solidity will be ·5236, and the superficies will be 3·1416; that is, as 1 to 6. And to find the superficial content of any sphere, multiply 3·1416 by the square of the diameter, and the product will be the superficial content. And for the solidity, multiply ·5236 by the cube of the diameter, the product will be the solid content; and, because ·5236 is a sixth part of 3·1416 ∴ 6 must be taken for the diameter. For, if 3·1416 be multiplied by the square of 6, *viz.* 36, the product will be 113·0976; and if ·5236 be multiplied by the cube of 6, *viz.* 216, the product is likewise 113·0976. Hence, the solidity is equal to the superficies which was required. ∴ 6 is the true *Answer.*

Ex. 32. Let D = the diameter, and C = the circumference. Then, (*see* Rule 2, Prob. XII. Solids.) The solidity of a sphere : its superficies :: $D \times C \times \frac{D}{6} : D \times C$.

$$\therefore 9 : 4 \left(:: \frac{D}{6} : 1\right) :: D : 6.$$

∴ 4 D = 54, and ∴ D = 13·5, the axis or diameter of the globe = *Answer.*

Ex. 33. If the diameter of a globe be 1, its solidity will be ·5236, and globes are in proportion to each other as the cubes of their diameters. Hence, 1 : 16^3 :: ·5236 : 2144·6656 solid inches, the solidity of the metal and concavity of the first shell, the half of which is 1072·3328 inches, the solidity of the metal of the first shell, and also *equals* the number of solid inches, which would fill its concavity. Then, $\frac{1072{\cdot}3328}{{\cdot}5236}$

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* Euclid, 4 p. vi. b.

= 2048 inches, the cube of the internal diameter of the first shell. Hence, $\sqrt[3]{2048}$ = 12·6992 inches, the internal diameter of the first shell *equals* the external diameter of the second shell = 1*st Answer.*

Now, 16 inches, the external diameter of the first shell — 12·6992 inches, the internal diameter of the first shell = 3·3008 inches; which ÷ by 2 = 1·6504 inches, the thickness of the metal of the first shell = *2nd Answer.*

Then, the solidity of the metal of the first shell = 1072·3328 cubic inches; which ÷ by 4 oz. = 268·0832 lbs. the weight of the metal of the first shell = *3rd Answer.*

Secondly. Since the second shell will just fill the concavity of the first, the external diameter of the second shell must be = 12·6992 inches, and the solidity of its metal and concavity together = 1072·3328 cubic inches. ∴ as 7 + 5 : 5 :: 1072·3328 cubic inches : 446·8053′ solid inches, which would fill the concavity of the second shell. Then, $\frac{446\cdot8053'}{\cdot5236}$ = 853·3′ inches, the cube of the internal diameter of the second shell. Hence, $\sqrt[3]{853\cdot3'}$ = 9·4850 inches, the internal diameter of the second shell *equals* the external diameter of the third shell = 4*th Answer.*

Now, 12·6992 inches, the external diameter of the second shell — 9·4850 inches, the internal diameter of the second shell = 3·2142 inches, which ÷ by 2 = 1·6071 inches, the thickness of the metal of the second shell = 5*th Answer.*

Then, 1072·3328 cubic inches, the solidity of the concavity of the first shell — 446·80533′ cubic inches, the solidity of the concavity of the second shell = 625·52746′ cubic inches, the content of the metal of the second shell. ∴ 625·52746′ cub. in. ÷ 4 oz. = 156·38186′ lbs, the weight of the metal of the second shell = 6*th Answer.*

Thirdly. Since the third shell will just fill the concavity of the second, its external diameter must be = 9·4850 inches, and the solidity of its metal and concavity together = 446·8053′ cubic inches. ∴ 9 + 4 : 4 :: 446·8053′ cub. in. : 137·4785′64102′5641025 cub. inches, which would fill the concavity of the third shell.

Then, $\frac{137\cdot4785'64102'5641025}{\cdot5236} = 262\cdot5'64102'564102$ inches, the cube of the internal diameter of the third shell. Hence, $\sqrt[3]{262\cdot5'64102'564102} = 6\cdot4034$ inches, the internal diameter of the third shell. Now, 9·4850 inches, the external diameter of the third shell − 6·4034 inches, the internal diameter of the third shell = 3·0816 inches; which ÷ by 2 = 1·5408 inches, the thickness of the metal of the third shell = *7th Answer.*

Then, 446·8053′ cubic inches, the solidity of the metal and concavity of the third shell − 137·478564 solid inches, which would fill the concavity of the third shell = 309·326769 cubic inches, the content of the metal of the third shell. ∴ 309·326769 cubic inches ÷ 4 oz. = 77·331692 lbs., the weight of the third shell = *8th Answer.*

Ex. 34. *(See Fig. 15 of this Key.) First.* Find the area of Will's pancakes, the versed sine, and the area of the segment. By Rule 2, Prob. XIII. Superficies. $DG^2 \times \cdot7854 = 15\cdot25^2 \times \cdot7854 = 182\cdot6545875$ sq. inches, the area of one of Will's pancakes ADBGA, which × by 4 = 730·61835 sq. inches, the area of Will's four pancakes. *$\sqrt{AC^2 - AF^2} = \sqrt{7\cdot625^2 - 6\cdot1^2} = \sqrt{20\cdot930625} = 4\cdot575$ inches = CF. Hence, CG − CF = 7·625 − 4·575 = 3·05 inches = FG, the versed sine of the segment of Will's pancake ABGA = *1st Answer.*

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* Euclid, 47 p. i. b.

Then, by Rule, Prob. XVII. Ex. 1. Superficies. FG ÷ DG = 3·05 ÷ 15·25 = ·200, the tabular height, against which stands the *Area. Seg.* ·111823. Hence, $\cdot 111823 \times 15{\cdot}25^2 = 26{\cdot}0058364375$ sq. inches, the area of the segment of Will's pancake ABGA = *2nd Answer.*

Second. Find the area of Jack's pancakes. By Rule 2, Prob. XIII. Superficies. $KH^2 \times \cdot 7854 = 12^2 \times \cdot 7854 = 113{\cdot}0976$ sq. inches, the area of one of Jack's pancakes HIKEH, which × by 6 = 678·5856 sq. inches, the area of Jack's six pancakes.

Now, 4 ADBGA − ABGA = 730·61835 − 26·0058364375 = 704·6125135625 sq. inches, Will's share = *3rd Answer.*

Again. 6 HIKEH + ABGA = 678·5856 + 26·0058364375 = 704·5914364375 sq. inches, Jack's share = *4th Answer.*

And ∴ 704·612513562 − 704·5914364375 = ·0210771245 sq. inches, Will had more than Jack after the decision of the gauger.

LINEAL MEASURE.

This measure is used to measure distances, lengths, breadths, heights, depths, &c. of places or things.

Inches.	Gunter's Link.	Foot.	Yard.	Rod, Pole, or Perch.	Gunters Chain.	Furlong.	Mile.
$7\frac{23}{25}$ =	= 1						
12	$1\frac{17}{33}$	= 1					
36	$4\frac{6}{11}$	3	= 1				
198	25	16½	5½	= 1			
792	100	66	22	4	= 1		
7920	1000	660	220	40	10	=1	
63360	8000	5280	1760	320	80	8	=1

Besides the above, there are, the Barley-corn, which equals ⅓ of an inch; the Palm = 3 inches; the Hand = 4 inches; the Span = 9 inches; the Cubit = 1½ foot; the common military Pace = 2 ft. 6 in.; the geometrical Pace = 5 feet; the Fathom = 6 feet; the League = 3 miles; and 60 geographical, or 69·114 English miles (according to Colonel Mudge) = 1 degree.

A Rope = 20 feet; a woodland Pole = 18 feet; a forest or plantation Pole = 21 feet; a Cheshire Pole = 24 feet; a Sherwood Forest Pole = 25 feet; and 1 rood of fencing or ditching = 7 yards.

SQUARE MEASURE.

Square measure is used to measure all kinds of superficies; such as land, paving, flooring, plastering, roofing, slating, tiling, and every thing that has length and breadth.

Sq. Inches.	Sq. Links	Sq. Foot.	Sq. Yard.	Sq Pole, Perch, or Rod.	Sq. chain	Sq. rood.	Sq. Acre.	Sq. Mile.
$62\frac{454}{625}$ =	= 1							
144	$2\frac{322}{1089}$	= 1						
1296	$20\frac{80}{121}$	9	= 1					
39204	625	$272\frac{1}{4}$	$30\frac{1}{4}$	= 1				
627264	10000	4356	484	16	= 1			
1568160	25000	10890	1210	40	$2\frac{1}{2}$	= 1		
6272640	100000	43560	4840	160	10	4	= 1	
4014489600	64000000	27878400	3097600	102400	6400	2560	640	= 1

A Hide of land = 100 acres; a Yard of land = 30 acres; a Square of flooring, partitioning, roofing, tiling, and slating, are each = 100 sq. feet; a Rod of brick-work = 272·25 sq. feet; in some places it is customary to allow 324 sq. feet to a Rod; sometimes a Rod = 21 feet long, and 3 feet high = 63 sq. feet.

CUBIC OR SOLID MEASURE.

This measure is used to measure all kinds of solids; or such figures as consist of three dimensions, *viz.* length, breadth, and depth, or thickness.

Cubic Inches.	Cubic Foot.	Cubic Yard.	Cubic Pole, Rod, or Perch.	Cub. Furlong.	Cub. Mile.
1728	= 1				
46656	27	= 1			
7762392	$4492\frac{1}{8}$	$166\frac{3}{8}$	= 1		
496793088000	287496000	10648000	64000	= 1	
254358061056000	147197952000	5451776000	32768000	512	= 1

A Load of rough timber = 40 cubic feet; a Load of squared timber = 50 cubic feet; a Ton of shipping = 42 cubic feet; a Floor of earth = 324 cubic feet; a Cord of wood = 8 ft. long. 4 ft. broad, and 4 ft. deep = 128 cubic feet; a Stack of wood = 12 ft. long 3 ft. broad, and 3 ft. deep = 108 cubic feet.

OLD WINE MEASURE.*

By this measure all wines, brandies, rums, spirits, distilled liquor, cyder, perry, mead, vinegar, honey, oil, &c. are measured, bought, and sold.

Imperial or New Gallons.	Gill.	Pint.	Quar.	Gall.	Ank.	Run-let.	Bar-rel.	Tier.	Hgs-head.	Pun.	Pipe.	Ton.
·0260416′	= 1											
·10416′	4	= 1										
·2083′	8	2	= 1									
†$\frac{5}{6}$ or =·83′	32	8	4	= 1								
8·3′	320	80	40	10	=1							
15·	576	144	72	18	$1\frac{4}{5}$	= 1						
26·25	1008	252	126	$31\frac{1}{2}$	$3\frac{3}{20}$	$1\frac{3}{4}$	= 1					
35·	1344	336	168	42	$4\frac{1}{5}$	$2\frac{1}{3}$	$1\frac{1}{3}$	=1				
52·5	2016	504	252	63	$6\frac{3}{10}$	$3\frac{1}{2}$	2	$1\frac{1}{2}$	=1			
70·	2688	672	336	84	$8\frac{2}{5}$	$4\frac{2}{3}$	$2\frac{2}{3}$	2	$1\frac{1}{3}$	1		
105·	4032	1008	504	126	$12\frac{3}{5}$	7	4	3	2	$1\frac{1}{2}$	1	
210.	8064	2016	1008	252	$25\frac{1}{5}$	14	8	6	4	3	2	1

OLD ALE AND BEER MEASURE.

By this measure all malt liquors are gauged, bought, and sold.

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* *Note.* At the alteration of the Old Measure to the Imperial (which took place Jan. 1st, 1826) the pint, quart, and gallon, in wine, ale, and beer, and grain or corn measure, are the same with regard to their magnitude: 8 of these gallons make one bushel (not measured by heaped measure); *one gallon* contains 277·274 cubic inches, or equals 10lbs. avoirdupoise of distilled water weighed in air at the temperature of 62 degrees of Fahrenheit's Thermometer, the Barometer being at 30 inches.

† To reduce Old Wine Gallons to Imperial, multiply the Old Wine Gallons by 1·2, and the product will be Imperial Gallons. CONVERSELY. To reduce Imperial Gallons to Old Wine Gallons, divide the Imperial Gallons by 1·2, and the quotient will be Old Wine Gallons. (*See* Article Gauging, Rules, wine measure, p. 112 *of this Key.*)

Imperial or New Gallons.	Pint.	Qt.	Gall.	Fir-kin-	Kil-der-kin.	Bar-rel.	Hgs. head.	Pun.	Butt.	Ton.
·127164	= 1									
·254329	2	= 1								
$1\frac{4}{231}$ or=1·017316	8	4	= 1							
9·155844	72	36	9	= 1						
18·311688	144	72	18	2	= 1					
36·623376	288	144	36	4	2	= 1				
54·935064	432	216	54	6	3	$1\frac{1}{2}$	= 1			
73·246753	576	288	72	8	4	2	$1\frac{1}{3}$	= 1		
109·870129	864	432	108	12	6	3	2	$1\frac{1}{2}$	= 1	
219·740259	1728	864	216	24	12	6	4	3	2	1

OLD DRY MEASURE.

This measure is used in measuring all dry commodities, as wheat, barley, beans, and other grain; fruit, roots, sand salt, coals, oysters, &c.

Imperial or New Gallons.	Pint.	Qt.	Pottl.	Gall.	Peck.	Bhl.	Strik.	Co.	Qr.	Way.	Last.
·1′2′	= 1										
·2′4′	2	= 1									
·4′8′	4	2	1								
$\frac{32}{33}$ or =·9′6′	8	4	2	= 1							
1·9′3′	16	8	4	2	= 1						
7·7′5′	64	32	16	8	4	= 1					
15·5′1′	128	64	32	16	8	2	= 1				
31·0′3′	256	128	64	32	16	4	2	= 1			
62·0′6′	512	256	128	64	32	8	4	2	= 1		
310·3′0′	2560	1280	640	320	160	40	20	10	5	= 1	
620·6′0′	5120	2560	1280	640	320	80	40	20	10	2	1

THE END.

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Fig. 16

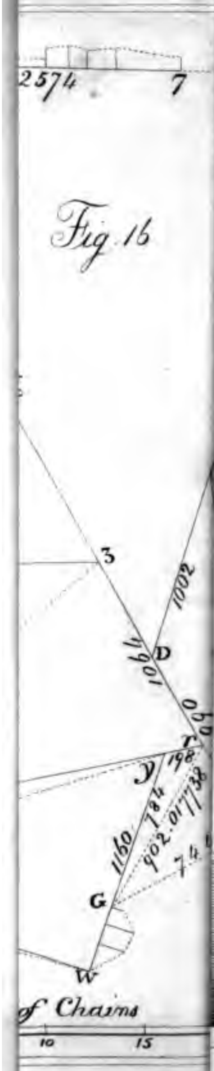
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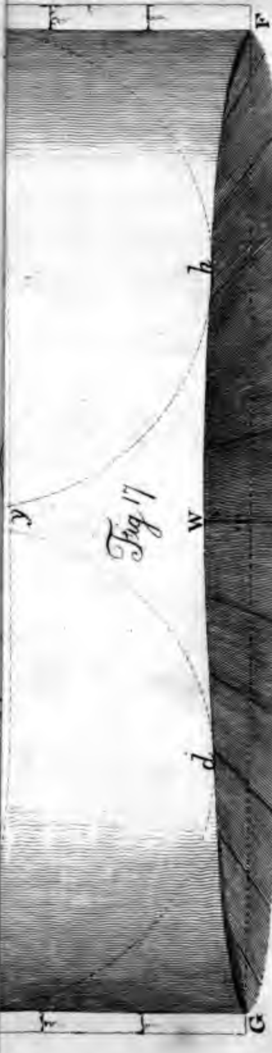
Fig. 17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